Math 1031  
Spring, 2006  
FINAL EXAM  

Name (Print)_________________________________  
Signature______________________________________  

T.A. Instructor________________________Discussion Section_______ I.D.#_______  

READ AND FOLLOW THESE INSTRUCTIONS  
This booklet contains 18 pages, including this cover page and the formula page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. Books, notes and graphing calculators are NOT PERMISSIBLE. Scientific calculator is NEEDED to answer some of the questions. Do your work in the blank spaces and back of pages of this booklet. Show all your work.  

There are 20 machine-graded problems worth 8 points each and 6 hand-graded problems worth 140 points together for a total of 300 points.  

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-20):  
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong.  

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 21-26):  
SHOW ALL WORK. Unsupported answers will receive little credit.  

Notice regarding the machine graded sections of this exam. Either the student or the School of Mathematics may for any reason request a regrading of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. Therefore work and answers must be clearly shown on the test booklet.  

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning in your exam.  

<p>| | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>26</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subtotal</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1-20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Letter Grade ____
Formulas in counting and probability

\[ P(n, k) = \frac{n(n-1)(n-2)\cdots(n-k+1)}{(n-k)!} = \frac{n!}{(n-k)!} = C(n, n-k) \]

\[ C(n, k) = \frac{n(n-1)(n-2)\cdots(n-k+1)}{k!} = \frac{n!}{k!(n-k)!} = C(n, n-k) \]

\[ P(E^C) = 1 - P(E) \]

\[ P(E \cup F) = P(E) + P(F) - P(E \cap F) \]

\[ P(E|F) = \frac{P(E \cap F)}{P(F)} \]

\[ P(E) = P(E|F)P(F) + P(E|F^C)P(F^C) \]

If we repeat \( n \) times a Bernoulli experiment for which the probability of success is \( p \) and the probability of failure is \( 1 - p \), then

\[ P(\text{exactly } k \text{ successes}) = C(n, k) p^k (1 - p)^{n-k}. \]

The expected value of a game with outcomes \( s_1, s_2, \ldots, s_n \) in which you win the amount \( w_i \) when outcome \( s_i \) occurs is

\[ E = w_1P(s_1) + w_2P(s_2) + \cdots + w_nP(s_n). \]

Formulas in compound interest

The balance \( A(t) \) in an investment account paying interest at an annual rate \( r \) (expressed in decimal) compounded \( n \) times per year, \( t \) years after making an initial deposit \( P \) is given by

\[ A(t) = P \left( 1 + \frac{r}{n} \right)^{nt}. \]

If the interest is compounded continuously, the balance is given by

\[ A(t) = Pe^{rt}. \]
1. Which answer describes the solution of $x^2 - 5x + 10 = 0$?

(a) No real solutions;
(b) One real solutions;
(c) Two real solutions;
(d) Three real solutions;
(e) All real numbers are solutions.

2. The distance between points $(-3, 2)$ and $(1, 4)$ is

(a) 6;
(b) $\sqrt{8}$;
(c) $\sqrt{20}$;
(d) $\sqrt{40}$;
(e) $\sqrt{52}$. 
3. The midpoint of the line segment joining the points \((5, 3)\) and \((-7, 1)\) is

(a) \((-1, 2)\);
(b) \((6, 1)\);
(c) \((-6, -1)\);
(d) \((-2, 4)\);
(e) \((12, 2)\).

4. You flip a fair coin five times. What is the probability of getting at least two heads?

(a) \(\frac{3}{5}\);
(b) \(\frac{2}{5}\);
(c) \(\frac{3}{32}\);
(d) \(\frac{26}{32}\);
(e) None of the above.
5. 10 fair dice are rolled, what's the probability of obtaining exactly six 6's?

   (a) \( \frac{6}{10} \);
   (b) \( \frac{1}{20} \);
   (c) \( \binom{10}{6} \frac{5^4}{6^{10}} \);
   (d) \( \binom{10}{6} \frac{5^6}{6^{10}} \);
   (e) \( \binom{10}{6} \frac{1}{6^5} \).

6. You flip four fair coins. What is the probability that all four show heads given that at least two are heads?

   (a) \( \frac{1}{16} \);
   (b) \( \frac{1}{11} \);
   (c) \( \frac{1}{8} \);
   (d) \( \frac{4}{11} \);
   (e) \( \frac{1}{4} \).
7. You have five used textbooks (in five different subjects) to give away and seven people are willing to take them. How many ways can you give them away to these people, if anyone can get any number of books?

(a) $7^5$;  
(b) $5^7$;  
(c) $C(7, 5)$;  
(d) $P(7, 5)$;  
(e) None of the above.

8. You play a game in which you flip two fair coins. If both are heads, you win 4 dollars. If both are tails you win 1 dollar. If one is head and the other tail, you lose 2 dollars. What is your expected winning in the game?

(a) Win 3 dollars;  
(b) Win 1 dollars;  
(c) Win 0.25 dollars;  
(d) Win 0.75 dollars;  
(e) Lose 3 dollars.
9. The center of the circle described by the equation
\[ x^2 + y^2 + 2x - 4y - 4 = 0 \]
is
(a) \((1, 2)\);
(b) \((-1, 2)\);
(c) \((1, -2)\);
(d) \((2, -4)\);
(e) \((0, -4)\).

10. The domain of function
\[ f(x) = \ln \frac{x + 1}{x - 2} \]
consists of all real numbers in the intervals
(a) \((-\infty, 2)\) and \((2, \infty)\);
(b) \((-2, 1)\);
(c) \((-\infty, -2)\) and \((1, \infty)\);
(d) \((-\infty, -1]\) and \((2, \infty)\);
(e) \((-\infty, -1)\) and \((2, \infty)\).
11. Which of the following line is perpendicular to 
\[ x - 2y + 2006 = 0 \]
(a) \( x - 2y + 1 = 0; \)
(b) \( x + 2y + 10 = 0; \)
(c) \( 2x - y + 100 = 0; \)
(d) \( 2x + y + 1000 = 0; \)
(e) None of the above.

12. The inverse of the function \( f(x) = \frac{x}{2x + 1} \) is 
(a) \( f^{-1}(x) = \frac{2x + 1}{x} \)
(b) \( f^{-1}(x) = -\frac{x}{2x - 1} \)
(c) \( f^{-1}(x) = -\frac{1}{2x - 1} \)
(d) \( f^{-1}(x) = -\frac{x}{2x + 1} \)
(e) None of the above.
13. Suppose you invest $500 at an annual rate of 5% compounded continuously. After \( t \) years you have $2000. Which correctly expresses \( t \)?

(a) \( \frac{\ln 4}{0.05} \);
(b) \( \frac{\ln 0.05}{4} \);
(c) \( \frac{4}{\ln 0.05} \);
(d) \( \frac{0.05}{\ln 4} \);
(e) None of the above.

14. The domain of function \( f(x) = \frac{\sqrt{2 - x^2}}{\sqrt{x}} \) is

(a) \((-\sqrt{2}, \sqrt{2})\);
(b) \((-\infty, -\sqrt{2}] \text{ and } [\sqrt{2}, \infty)\);
(c) \((-\sqrt{2}, 0) \text{ and } (0, \sqrt{2})\);
(d) \([-\sqrt{2}, 0) \text{ and } (0, \sqrt{2}]\);
(e) \([-\sqrt{2}, \sqrt{2}]\).
15. Assume \( x > 1 \), then

\[
\ln \left( \frac{x - 1}{(x + 1)^2} \right) - \ln \sqrt{x - 1} + \ln (x + 1)
\]

can be simplified to:

(a) \( -\frac{1}{2} \ln x - \frac{3}{2} \);
(b) \( \frac{1}{2} \ln (x - 1) - \ln (x + 1) \);
(c) \( \frac{1}{2} \ln (x - 1) + 3 \ln (x + 1) \);
(d) \( -\ln (x - 1) - \ln (x + 1) \);
(e) None of the above.

16. The \( x \)-intercept and \( y \)-intercept of \( 3x - 2y - 9 = 0 \) is

(a) \((3, 0)\) and \((0, \frac{9}{2})\);
(b) \((-3, 0)\) and \((0, -\frac{9}{2})\);
(c) \((3, 0)\) and \((0, -\frac{9}{2})\);
(d) \((-3, 0)\) and \((0, \frac{9}{2})\);
(e) None of the above.
17. In a probability model, event $E$ has probability $\frac{1}{3}$, event $F$ has probability $\frac{1}{2}$, assuming $E$ and $F$ are independent, the probability of $E \cup F$ is.

(a) $\frac{1}{6}$;
(b) $\frac{2}{6}$;
(c) $\frac{3}{6}$;
(d) $\frac{4}{6}$;
(e) $\frac{5}{6}$.

18. Let function $f(x) = x - \frac{1}{2}$ and $g(x) = x^2 - 1$, then $(f \circ g)(2)$ is equal to

(a) $\frac{8}{3}$;
(b) $\frac{8}{4}$;
(c) 3;
(d) $-\frac{1}{3}$;
(e) None of the above.
19. If we shift the graph of \( f(x) = x^2 - x \) to the left 2 units, we obtain the graph of:

(a) \( x^2 + 3x + 2 \);
(b) \( x^2 - 5x + 6 \);
(c) \( x^2 + 3x + 4 \);
(d) \( x^2 - x + 2 \);
(e) \( x^2 - x - 2 \).

20. In order to obtain the graph \( y = \ln(x + 1) + 2 \), we should shift the graph \( y = \ln x \)

(a) to the left by 1 unit and upward by 2 units;
(b) to the right by 1 unit and upward by 2 units;
(c) to the left by 2 unit and upward by 1 units;
(d) to the right by 2 unit and upward by 1 units;
(e) None of the above.

End of multiple-choice problems
21. (24 points) A certain club has 5 males and 4 female members.

   (a) (12 points) How many ways are there to form a 4 person committee which has at least one male and one female?

   (b) (12 points) How many ways are there to form a 4 person committee if two certain members refuse to be in the committee at the same time?
22. (24 points) Find the solution of the following equations:

(a) (12 points) \[ e^{2x} + 5e^x - 6 = 0; \]

(b) (12 points) \[ x - 1 = \sqrt{x + 1}. \]
23. (24 points) Solve the following inequalities:

(a) (12 points) \[
\frac{2x + 1}{x - 1} \leq 1;
\]

(b) (12 points) \[
|2x - 3| > 1.
\]
24. (30 points) Let 

\[ f(x) = -2x^2 + 3x - 1. \]

(a) (5 points) Rewrite the function in standard form.
(b) (5 points) Find \( x \)-intercept and \( y \)-intercept of its graph.
(c) (5 points) Find the vertex of the graph.
(d) (5 points) Sketch the graph of the function.
(e) (5 points) For what values of \( x \) does the function increase?
(f) (5 points) Find the domain and range of this function.
25. (18 points) The profit (in dollar) for a company is given by

\[ P(x) = -0.0001x^2 + 100x - 200,000 \]

where \( x \) is the number of units produced. What is the number of units that should be produced to yield a maximum of profit?
26. (20 points) You invest $5,000 in an account with annual interest rate of 4%. Find the balance after 10 years if the interest is

(a) (10 points) compounded quarterly;
(b) (10 points) compounded continuously.