Math 1031. Spring 2009
FINAL EXAM

Name:............................Signature:............................

TA:............................Discussion:.........................ID:............

READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 16 pages. Check to see if any are missing.
PRINT all the requested information, and sign your name. Put your name on the top of
every page, in case the pages become separated. Books and notes are not permissible. Only
scientific calculators are allowed. Do your work in the blank spaces and back of pages of this
booklet. Show all your work.
There are 20 machine-graded problems, worth 5 points each. There are 5 hand-graded problems,
with point values listed at the problem, making a total score of 200 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1–20):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the
answer sheet, and carefully enter all the requested information according to the instructions
you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a
correct answer to a given question, circle the answer in this booklet
and blacken completely the corresponding circle in the answer sheet. If you erase something,
do so completely. Each question has a correct answer. If you give two different answers, the
question will be marked wrong. There is no penalty for guessing, but if you don't answer a
question, skip the corresponding line in the answer sheet. Go on to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 21–25):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the School
of Mathematics may for any reason request a regrade of the machine graded part. All regrades
will be based on responses in the test booklet, and not on the machine graded response sheet.
Any problem for which the answer is not indicated in the test booklet, or which has no relevant
accompanying calculations will be marked wrong on the regrade. Therefore work and answers
must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM; Place the answer sheet between two pages of this booklet
(make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing DOWN. Have your
ID card in your hand when turning in your exam.

Machine-graded part:................Hand-graded part:................Total:..............
(200 points max)

| 21 (20) | 22 (15) | 23 (20) | 24 (20) | 25 (25) | subtotal (100) |
Some formula reminders
Note: These formulas are intended to help you recall some topics in the course, but they are not complete or comprehensive. You will need to know other formulas besides the ones given here, and some formulas given here may not be used in the exam.

Interest: Compounding interest \( n \) times per year: \( A = P \left( 1 + \frac{r}{n} \right)^{nt} \)

Continuous Compounding: \( A = Pe^{rt} \)

Equally likely outcomes: \( P(E) = \frac{n(E)}{n(S)} \)

Independence (intersections): \( P(E \cap F) = P(E) \cdot P(F) \)

Unions: \( P(E \cup F) = P(E) + P(F) - P(E \cap F) \)

Complements: \( P(E') = 1 - P(E) \)

Conditional Probability: \( P(E|F) = \frac{P(E \cap F)}{P(F)} \) or \( P(E \cap F) = P(F) \cdot P(E|F) \)

Expected Value: \( E_v = x_1P_1 + x_2P_2 + ... + x_kP_k \)

Binomial Probability: \( P(\text{successes}) = C(n,x)p^x(1-p)^{n-x} \)
Part I, 20 Machine-Graded Problems, worth 5 points each

Problem 1: A rectangular plot of ground measuring 9 meters by 14 meters is surrounded by a sidewalk of uniform width. The area of the sidewalk is 330 square meters. How wide is the sidewalk?

a) 2 meters
b) 3 meters
c) 4 meters
d) 5 meters
e) none of the above

Problem 2: If (3, -5) and (1,13) are the endpoints of a diameter of a circle, what is the center of the circle?

a) (-4, 3)
b) $(2\sqrt{34}, 5)$
c) (0,0)
d) (2,4)
e) (2,12)
Problem 3: Find the linear function whose graph is a line with y-intercept -8 and which is perpendicular to the line: \( f(x) = \frac{4}{3}x + 2 \).

a) \( g(x) = -\frac{4}{3}x - 8 \)

b) \( g(x) = -\frac{3}{4}x - 8 \)

c) \( g(x) = \frac{3}{4}x - 12 \)

d) \( g(x) = \frac{4}{3}x + 8 \)

e) \( g(x) = -\frac{3}{4}x + 2 \)

Problem 4: Determine the type of symmetry possessed by the graph of the equation: \( 4xy + x^2 + y^2 = 3 \)

a) x- axis

b) y- axis

c) x- axis and y- axis

d) origin

e) all of the above
Problem 5: Find the domain of the function:

\[ f(x) = \frac{2}{x} - \sqrt{x - 3} \]

a) \([3, \infty)\)
b) \(\{x \mid x \neq 0\}\)
c) \((0, 3) \cup (3, \infty)\)
d) \((3, \infty)\)
e) \((-\infty, 0) \cup (0,3)\)

Problem 6: Solve the equation: \(\sqrt{5x + 31} - \sqrt{x + 3} = 4\)

a) \(x = -3\)
b) \(x = -1\)
c) \(x = 1\)
d) \(x = -3\) or \(x = 13\)
e) \(x = -3\) or \(x = 1\)
Problem 7: Given the function: \( f(x) = 2 \sqrt{x - 3} + 5 \)
How has the graph of \( g(x) = 2\sqrt{x} \) been shifted?

a) right 3 and up 5  
b) left 3 and down 5  
c) right 3 and down 5  
d) left 3 and up 5

Problem 8: If \( f(x) = \sqrt{5 + x} \) and \( g(x) = x^2 - 5 \), Then \( f \circ g(-2) = \)

a) -2  
b) 2  
c) 4  
d) not a real number

d) not a real number

Problem 9: The inverse function of the function: \( f(x) = (x + 2)^2 \) is

a) \( f^{-1}(x) = \frac{1}{(x+2)^2} \)  
b) \( f^{-1}(x) = \sqrt{x + 2} \)  
c) \( f^{-1}(x) = \sqrt{x} - 2 \)  
d) \( f^{-1}(x) = \sqrt{x} - 2 \)  
e) There is no inverse function
Problem 10: Which of the following is equal to \( \log (15^3) \)

a) \( \log 3 + \log 15 \)  
b) \( 3(\log 5 + \log 3) \)  
c) \( \log 5 + 3 \log 3 \)  
d) \( (3 \log 5)(\log 3) \)  
e) \( \log 9 + \log 5 \)

Problem 11: A ball is thrown straight upwards so that its height in feet above ground \( t \) seconds after being thrown is \( h(t) = -16t^2 + 64t \).  
What is the maximum height reached by the ball?

a) 2 feet  
b) 64 feet  
c) 128 feet  
d) 64 feet  
e) none of the above

Problem 12: What is the end behavior of the following function? \( f(x) = -3x^5 + 4x^4 + 7x^3 \)

a) left end goes up, right end goes up  
b) left end goes up, right end goes down  
c) left end goes down, right end goes down  
d) left end goes down, right end goes up
Problem 13: Solve: \((2^a)(2^{8a}) = 32\)

a) \(a = \frac{9}{5}\)
b) \(a = \frac{5}{9}\)
c) \(a = \frac{16}{9}\)
d) \(a = 1\)
e) none of the above

Problem 14: Thirty students signed up to play summer basketball at a summer camp. How many different six person teams are possible? (5 players and one substitute on the team)

a) 216
b) 6
c) 1,947,792
d) 1,402,410,240
e) none of the above
Problem 15: You have three marbles, colored red, blue and green. You also have six cups numbered one through six. How many ways can you place the marbles into the cups, if no cup can hold more than one marble?

a) 18  
b) 20  
c) 120  
d) 216  
e) 729

Problem 16: The combination for a combination lock consists of a sequence of three numbers, using the numbers 0 through 59. How many possible unique combinations are possible if no numbers can be repeated?

a) 216,000  
b) 205,320  
c) 180  
d) 177

Problem 17: Draw two marbles (without replacement) from a bag containing one green, two yellow, and three red marbles. What is the probability of drawing exactly one red marble in the two draws?

a) 2/5  
b) 0  
c) 3/5  
d) 11/15
Problem 18: In how many distinguishable ways can the letters in the work BANANA be written?

a) 720  
b) 1296  
c) 6  
d) 60

Problem 19: For a fee of $2.00, you can play a game in which two fair coins are tossed. If the result of the two coin toss is two heads, you win $5; if one coin is a head and one is a tail, you win $1; If both are tails, you win nothing. What is your expected gain or loss from this game?

a) -$0.25  
b) $0.25  
c) $1.00  
d) $1.25  
e) none of these

Problem 20: A card is drawn at random from a standard deck of cards. What is the probability that the card is a club or an even numbered card? (The numbered cards are the ace through the 10.)

a) 33/52  
b) 38/52  
c) 28/52  
d) 20/52
Part II  5 hand-graded problems. Points are assigned at each problem.

Problem 21: Consider the circle given by the equation:
\[ x^2 + y^2 + 10x - 8y + 32 = 0 \]

a) (12 points) Write the standard form of this circle

b) (3 points) What are the radius and the center of the circle?

c) (5 points) Graph this circle:
Problem 22: (15 points)
Solve the inequality:
\[(2x-1)(x-3)(x+5) \geq 0\]
Problem 23:
a) (10 points)
Solve the equation:
\[
\frac{4}{x-2} + \frac{x}{x+1} = \frac{x^2}{x^2-x-2}
\]

b) (10 points)
Solve the equation:
\[
\log_2 (x-4) + \log_2 x = 3
\]
Problem 24: Consider the expression: \( f(x) = 2|x - 1| - 2 \)

a)(4 points)
Find the x-intercepts

b)(2 points)
Find the y-intercepts

c)(6 points)
Sketch the graph

![Graph](image)

d)(4 points)
Using the graph, what is the domain and range:

Domain:

Range:

e)(4 points)
Using the graph, where is \( f(x) \) increasing, decreasing?
Problem 25: The experiment is to draw a card from a standard deck, four times, replacing and reshuffling after each draw.

a) (3 points)
What is the size of the sample space of this experiment?

b) (5 points)
What is the probability of drawing exactly 4 aces?

c) (5 points)
What is the probability of drawing exactly 2 aces?

d) (6 points)
What is the probability of drawing at least 2 aces?

e) (6 points)
What is the probability of drawing at most 2 aces?
Use this page if you need extra space to show your work. Please indicate problem number.