Math 1031
Final Exam, Spring 2011

Name:

Discussion Section:

Discussion Instructor:

You may use a scientific calculator, but you may not use books, notes, graphing calculators, or your neighbors' papers. Sign your name below to certify that you followed these instructions.

Signature:

Do all your work in the space provided on these sheets. If you need additional paper, attach it to these sheets.

Directions for machine graded part: You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing, but if you do not answer a question, skip the corresponding line in the answer sheet.

Notice regarding the machine graded sections of this exam: Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations, will be marked wrong on the regrade. Therefore work and answers must be clearly shown on the test booklet.

Directions for hand graded part: Partial credit will be rewarded on the short answer problems. You will not earn credit for illogical, incorrect, or unsupported work, even if you miraculously arrive at the correct answer. If you are not certain how to do a problem, give it your best attempt so that you may earn some credit for moving in the right direction. Circle your final answer on the short answer problems.

The exam will be graded out of 200 points. The point value for each problem is listed beside the problem number. There are 17 pages and 25 problems on the exam.

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
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<tbody>
<tr>
<td>1-20</td>
<td>(120)</td>
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<td>21</td>
<td>(18)</td>
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<td>25</td>
<td>(18)</td>
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<td>Total</td>
<td>(200)</td>
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Here are some formulas that you might find useful.

\[ A = P(1 + r)^t \]

\[ A = P \left(1 + \frac{r}{n}\right)^{nt} \]

\[ A = Pe^{rt} \]

\[ P(E) = \frac{n(E)}{n(S)} \]

\[ P(E') = 1 - P(E) \]

Independence: \( P(E \cap F) = P(E) \cdot P(F) \)

\[ P(E \cup F) = P(E) + P(F) - P(E \cap F) \]

\[ P(x \text{ successes}) = C(n, x)p^x(1 - p)^{n-x} \]

\[ P(E|F) = \frac{P(E \cap F)}{P(F)} \quad \text{or} \quad P(E \cap F) = P(F) \cdot P(E|F) \]

\[ E_o = x_1p_1 + x_2p_2 + \ldots + x_np_n \]
1. (6 points) Find all solutions of the following equation.

\[
\frac{x}{x - 1} + \frac{3}{x - 2} = \frac{-1}{x^2 - 3x + 2}
\]

(a) \{-2\}
(b) \{-2, 1\}
(c) \{-1\}
(d) \{-1, 2\}

2. (6 points) Simplify the following expression.

\[
\left(\frac{xz}{x^3y^{-2}}\right)^3
\]

(a) \(\frac{y^5z^4}{x^5}\)
(b) \(\frac{y^6z^3}{x^6}\)
(c) \(\frac{z^3}{(\sqrt{y})^3x^6}\)
(d) \(\frac{z^4}{(\sqrt{y})^3x^5}\)
3. (6 points) Find all the roots (also called zeroes) of the function

\[ f(x) = 2x^3 - 7x^2 - 9x. \]

(a) \((-3/2, 0), (3, 0)\)
(b) \((-1, 0), (9/2, 0)\)
(c) \((-3/2, 0), (0, 0), (3, 0)\)
(d) \((-1, 0), (0, 0), (9/2, 0)\)

4. (6 points) Simplify the following expression.

\[ \sqrt{x^2 - 1}y^5 \]

(a) \((x - 1)\sqrt{y^5}\)
(b) \((x - 1)y^2\sqrt{y}\)
(c) \(y^4\sqrt{(x^2 - 1)y}\)
(d) \(y^2\sqrt{(x^2 - 1)y}\)
5. (6 points) You begin with 10 ounces of salad dressing which is 60% oil and 40% vinegar. How much oil should you add to obtain a mixture which is 75% oil and 25% vinegar?

(a) 10 ounces
(b) 8.5 ounces
(c) 6 ounces
(d) 1.5 ounces

6. (6 points) Find \( f(0) \) and \( f(2) \), if \( f(x) \) is the piecewise function given below.

\[
f(x) = \begin{cases} 
  x^2 & \text{if } x \geq 2 \\
  x - 3 & \text{if } x < 2 
\end{cases}
\]

(a) \( f(0) = 0, f(2) = 4 \)
(b) \( f(0) = 0, f(2) = -1 \)
(c) \( f(0) = -3, f(2) = 4 \)
(d) \( f(0) = -3, f(2) = -1 \)
7. (6 points) Find the domain of the function \( f(x) = \sqrt{x^2 + 2x - 8} \).

(a) \( \{ x : x \leq -4 \text{ or } x \geq 2 \} \)
(b) \( \{ x : -4 \leq x \leq 2 \} \)
(c) \( \{ x : x \neq -4, x \neq 2 \} \)
(d) \( \{ x : x = -4 \text{ or } x = 2 \} \)

8. (6 points) Find the center and radius of the following circle.

\[ x^2 + y^2 - 6x + 10y + 30 = 0 \]

(a) center \((-3, 5)\), radius 4
(b) center \((3, -5)\), radius 4
(c) center \((-3, 5)\), radius 2
(d) center \((3, -5)\), radius 2
9. (6 points) Which of the following graphs is symmetric about the origin?

(i) $y = -(x^3)$
(ii) $y = x^3 + 4$
(iii) $y = x^3 + 4x$

(a) (i) only
(b) (i) and (iii) only
(c) (i), (ii), and (iii)
(d) none of the graphs

10. (6 points) You are 30 miles east of St. Paul at time $t = 0$. You are driving east at 60 miles per hour. Find a function $d(t)$ that gives your distance from St. Paul in miles, in terms of time $t$ measured in hours.

(a) $d(t) = 60t$
(b) $d(t) = 1800t$
(c) $d(t) = 30 - 60t$
(d) $d(t) = 30 + 60t$
11. (6 points) For the functions $f(x) = 4x - 5$ and $g(x) = \frac{2x}{x - 2}$, find $(f \circ g)(3)$.

(a) 19 

(b) 42 

(c) $\frac{24x}{x - 2} - 15$ 

(d) $\frac{24x^2 - 30x}{x - 2}$ 

12. (6 points) Suppose $f(x) = |x|$ is scaled by a factor of 4, reflected over the $x$-axis, and translated right 6 units. What is the resulting function?

(a) $f(x) = 4| - x + 6|$ 
(b) $f(x) = 4| - x - 6|$ 
(c) $f(x) = -4|x + 6|$ 
(d) $f(x) = -4|x - 6|$
13. (6 points) You invest $7,000 at 5% interest compounded 4 times per year. How much money will have accumulated after 3 years?

(a) $7,265.79 
(b) $7,356.62 
(c) $8,103.38 
(d) $8,125.28 

14. (6 points) Solve for $x$ in this logarithmic equation:

$$\log_2 x + \log_2 (x - 3) = 2$$

(a) $x = \{4\}$
(b) $x = \{4, -1\}$
(c) $x = \{7/2\}$
(d) $x = \{7/2, 1/2\}$
15. (6 points) Write this expression as a single logarithm:
\[
\frac{1}{2} \ln(x - 2) + 5 \ln y - \ln(3z)
\]
(a) \(\ln \left( \sqrt{x - 2}(y^5)(-3z) \right)\)
(b) \(\ln \left( \frac{1}{2}(x - 2)(5y)(-3z) \right)\)
(c) \(\ln \left( \frac{\sqrt{x - 2}(y^5)}{3z} \right)\)
(d) \(\ln \left( \frac{\frac{1}{2}(x - 2)(5y)}{3z} \right)\)

16. (6 points) Five cars are driving in a line. They are brown, blue, yellow, red, and black. How many different lines can they form if the first car is red or blue and the last car is not black?

(a) 36
(b) 48
(c) 8
(d) 6
17. (6 points) We toss 2 dice. What is the probability that we get a sum of at least 4?

(a) $\frac{5}{6}$
(b) $\frac{8}{9}$
(c) $\frac{11}{12}$
(d) $\frac{17}{18}$

18. (6 points) There are 7 people in a club. In how many ways can they choose president, vice president, and secretary?

(a) 35
(b) 210
(c) 343
(d) 6
19. (6 points) A CEO makes a business deal which will cause the company to earn $10,000 with 15% probability or lose $500 with 85% probability. Find the expected value of this business deal.

(a) $1,000
(b) $1,075
(c) $1,925
(d) $9,500

20. (6 points) In a carton of 12 eggs, the probability that any one egg will be broken is 2%. What is the probability that 2 or more eggs will be broken in a carton of 12 eggs?

(a) 0.2%
(b) 2.0%
(c) 2.3%
(d) 4.0%
21. (18 points) Reminder: Give exact values, not decimal approximations.

(a) Find the distance between the points \((5, -1)\) and \((-2, -4)\).

(b) Find the equation of the line which passes through the points \((5, -1)\) and \((-2, -4)\). Write the line equation in slope intercept form.

(c) Find the equation of the line which goes through the point \((-1, -1)\) and is perpendicular to the line found in part (b). Write the line equation in slope intercept form.
22. (14 points) The height of a ball at time $x$ is given by

$$f(x) = -x^2 + 4x + 12,$$

where $x$ is measured in seconds and height $f(x)$ is measured in feet.

(a) Complete the square to write $f(x)$ in the form $f(x) = a(x - h)^2 + k$.

(b) Graph the function $f(x) = -x^2 + 4x + 12$. Label the axes, the intercepts, and the vertex.

(c) At what time $x$ does the ball reach its maximal height?

(d) What is the maximal height the ball reaches?
23. (12 points)

(a) Solve the inequality. Express your solution in interval notation.

\[ \frac{1}{2} |3x + 1| - 3 \geq 4 \]

(b) Plot your solution on a number line.
24. (18 points)

(a) Graph the function \( f(x) = 2^{x+1} \). Label the axes, intercepts, and two additional points on the graph.

(b) Does \( f(x) = 2^{x+1} \) have an inverse? If it does not have an inverse, explain why not. If it does, find its inverse.
25. (18 points) A hospital has 200 patients who have a certain disease. The data in the following table gives the number of patients who were administered an experimental drug and the number of patients who recovered from the disease.

<table>
<thead>
<tr>
<th></th>
<th>Drug Administered (D)</th>
<th>Drug Not Administered (D')</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Patient Recovered (R)</td>
<td>93</td>
<td>12</td>
<td>105</td>
</tr>
<tr>
<td>Patient Did Not Recover (R')</td>
<td>27</td>
<td>68</td>
<td>95</td>
</tr>
<tr>
<td>Total</td>
<td>120</td>
<td>80</td>
<td>200</td>
</tr>
</tbody>
</table>

(a) Find the probability that a patient recovered or that the experimental drug was administered to him or her. Give both your solution and the formula you used to find it.

(b) Find the probability that a patient recovered given that the experimental drug was administered to him or her. Give both your solution and the formula you used to find it.

(c) Let R be the event "the patient recovered," and let D be the event "the patient was administered the drug." Are the events R and D independent or dependent? Support your answer with specific calculations.