READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 11 pages, including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. Textbooks, notes and note cards are not allowed. Only scientific calculators are allowed. Calculators with graphing or symbolic manipulation capabilities are prohibited. Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 15 machine-graded problems worth \(\frac{1}{2}\) points each and 5 hand-graded problems worth 24 points each for a total of 300 points. **#4 not graded.**

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-15):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. **Do not make any stray marks on the answer sheet.** When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 16-20):
SHOW ALL WORK. You must show all steps in your solutions and make your reasoning clear to earn credit.

NOTICE REGARDING THE MACHINE GRADED PORTION OF THIS EXAM:
Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. Therefore work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning in your exam.

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Total ______

Letter Grade ______
1. If \( f(x) = \ln x \), then \( \lim_{h \to 0} \frac{f(2 + h) - f(2)}{h} \)

(a) \( = \ln 2 \)
(b) \( = 0 \)
(c) \( = \frac{1}{2} \)
(d) \( = 2 \ln 2 \)
(e) does not exist

2. \( \int_0^1 x^2(x^4 + 2)^2 \, dx = \)

(a) \( \frac{1}{12} \)
(b) \( \frac{19}{12} \)
(c) \( \frac{139}{12} \)
(d) \( \frac{1}{3} \)
(e) \( -\frac{1}{12} \)

3. What is the slope of the tangent line to the curve \( y = e^{x^2} + 2x^3 + 1 \) at the point where \( x = 2 \)?

(a) \( \frac{3}{2} \)
(b) \( e^4 + 24 \)
(c) \( e^3 + 8 \)
(d) \( 4e^4 + 24 \)
(e) \( \ln 8 \)
4. The domain of the function \( f(x) = \frac{x^3}{\sqrt{3x - 2}} \) is

(a) \( x > \frac{2}{3} \)

(b) \( x \geq \frac{2}{3} \)

(c) \( x \neq \frac{2}{3} \)

(d) \( x \leq \frac{2}{3} \)

(e) \( x < \frac{2}{3} \)

5. The derivative of \( f(x) = \frac{3x}{4x^2 + 5} \) is

(a) \( \frac{3}{8x^2} \)

(b) \( \frac{15 - 12x^2}{(4x^2 + 5)^2} \)

(c) \( \frac{12x^2 - 15}{(4x^2 + 5)^2} \)

(d) \( \frac{-12}{(4x^2 + 5)^2} \)

(e) none of the above

6. \( y^2 - 4xy + 3x = 0 \) describes a curve in the plane. At the point on the curve where \( x = 1 \)
and \( y = 1 \), the slope of the tangent line to the curve is

(a) \( -\frac{1}{2} \)

(b) \( \frac{1}{2} \)

(c) 1

(d) 0

(e) none of the above
7. \[ \lim_{{x \to 2}} \frac{x^3 - x^2 - 2x}{x^2 - 4} = \]

(a) \( +\infty \)

(b) \( -\infty \)

(c) 2

(d) 3/2

(e) does not exist

8. \[ \lim_{{x \to -\infty}} \frac{-x^2 + 3x + 5}{4x^3 - 2x^2 + 8x - 2} = \]

(a) \( -\infty \)

(b) \( \frac{1}{4} \)

(c) \( -\frac{1}{4} \)

(d) 0

(e) \( +\infty \)

9. For \( f(x) = \begin{cases} \frac{x-4}{\sqrt{x-2}} & \text{when } x > 4 \\ \frac{3}{2}x - 2 & \text{when } x \leq 4 \end{cases} \), which of the following is NOT true?

(a) The function is continuous at \( x = 3 \).

(b) The function is not continuous at \( x = 4 \).

(c) \( f(4) = 4 \)

(d) \( \lim_{{x \to 4}} f(x) \) exists

(e) \( \lim_{{x \to 4}} f(x) = f(4) \)
10. For $f(x) = e^{-x^2}$, which of the following is NOT true?

(a) $f(x)$ is increasing for $x > 0$
(b) $f(x)$ is concave up for all $x$
(c) $f(x)$ is decreasing for $x < 0$
(d) $f(x)$ is concave down for $x < 0$
(e) $f(x)$ is concave up for $x > 0$

11. $\int_1^\infty \frac{1}{(2x + 1)^3} \, dx$

(a) converges and its value is $-\frac{1}{2}(2x + 1)^{-2} + C$
(b) converges and its value is $\frac{1}{18}$
(c) converges and its value is $-\frac{1}{36}$
(d) converges and its value is $\frac{1}{36}$
(e) diverges

12. The area of the region between the curves $y = x^2$ and $y = -2x^3$ from $x = 0$ to $x = 2$ is

(a) $-\frac{15}{3}$
(b) $-\frac{32}{3}$
(c) $\frac{16}{3}$
(d) $-\frac{19}{3}$
(e) $\frac{32}{3}$
13. If \( f(x, y) = \frac{xy}{y^2 + 2} \), then \( f_x(3, 2) \) is

(a) \(-\frac{1}{3}\)

(b) \(\frac{1}{3}\)

(c) \(-\frac{1}{6}\)

(d) \(\frac{3}{11}\)

(e) none of the above

14. The function \( f(x, y) = 4x^3 + y^3 - 6x^2 - 6y^2 + 5 \) has a

(a) relative minimum at \((0, 0)\)

(b) relative maximum at \((1, 4)\)

(c) relative maximum at \((0, 4)\)

(d) saddle point at \((1, 0)\)

(e) saddle point at \((0, 0)\)

15. The average value of \( f(x) = x^2 - 3 \) over the interval \(-1 \leq x \leq 1\) is

(a) \(-5\frac{1}{3}\)

(b) \(-\frac{8}{3}\)

(c) \(\frac{16}{3}\)

(d) \(-\frac{16}{3}\)

(e) 4
16. (24 points) Find the maximum and minimum values of \( f(x) = 2x^3 - 3x^2 - 12x + 1 \) on the interval \(-2 \leq x \leq 0\).
17. (a) (12 points) Use integration by parts to evaluate \( \int (x + 1)e^{2x} \, dx \).

(b) (12 points) Use the substitution method to find \( \int \frac{x^2 + 2}{x^3 + 6x} \, dx \).
18. (24 points) A closed rectangular box (i.e. one with a top) is to have surface area 1200 sq. inches, and the longer side of the base is to be twice as long as the shorter side. Find the dimensions of the box, and its volume, if the volume is to be maximal.
19. (24 points) Find the unique function \( y = f(x) \) satisfying

\[
\frac{dy}{dx} = x^3 y^2 \quad \text{and} \quad y = \frac{1}{2} \quad \text{when} \quad x = 0
\]
20. (24 points) If 5,000 dollars is deposited in a certain savings account with continuous compounding, it will take 10 years for this initial investment to grow to 10,000 dollars. How long it will take for this initial investment of 5,000 dollars to grow to 15,000 dollars?