Math 1142
Final Exam
Spring 2008

Print Name__________________________________________
Signature____________________________________________

Question 0:
My TA’s name is __________________________________ and my section number is ______

This booklet contains 11 pages, including this cover page. Check to see if any are missing. Fill out the information requested above and put your initials on the top of every page, in case the pages become separated.

Books and notes are not permissible. Only scientific calculators are allowed.

There are 12 multiple choice questions worth 12 points each and six hand-graded questions worth varying points as shown for a total of 300 points.

Instructions for machine graded part: You MUST use a soft pencil to answer this part. Do not fold or tear the answer sheet, and carefully enter all requested information according to the instructions you receive. Do not make any stray marks on the answer sheet. Mark your answer to each question on the answer sheet and also circle your answer in the exam booklet. In case of machine error, we will use this booklet to reevaluate your answers, so you must circle your answer in this booklet, as well as filling out the answer sheet.

Instructions for the hand-graded part: Show all work. Unsupported answers will receive little credit.

After you finish both parts of the exam, place the answer sheet between two pages of this booklet (making a sandwich), with the side marked General Purpose Answer Sheet facing DOWN. Have your ID card ready when you hand on the exam.

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1. Let $f(x)$ be a function whose first derivative is $f'(x) = (x-2)(x-3)$. Then we can be sure that
(a) $f$ is increasing on the interval $(2,3)$
(b) $f$ is decreasing on the interval $(2,3)$
(c) $f$ is concave up on the interval $(2,3)$
(d) $f$ is concave down on the interval $(2,3)$
(e) none of the above

2. Let $f(x)$ be a function whose second derivative is $f''(x) = (x-2)(x-3)$. Then we can be sure that
(a) $f$ is increasing on the interval $(2,3)$
(b) $f$ is decreasing on the interval $(2,3)$
(c) $f$ is concave up on the interval $(2,3)$
(d) $f$ is concave down on the interval $(2,3)$
(e) none of the above

3. For what value or values of the constant $A$ is the following function continuous for all real $x$?

$$f(x) = \begin{cases} 
  x^2, & \text{if } x < 1 \\
  x + A, & \text{if } 1 \leq x \leq 2 \\
  x^3, & \text{if } x > 2
\end{cases}$$

(a) $A = 0$
(b) $A = 6$
(c) $A = 0$ and $A = 6$
(d) no value of $A$
(e) none of the above

4. The derivative of $\sqrt{x^3+1}$ is

(a) $\frac{1}{\sqrt{x^3+1}}$
(b) $\frac{1}{2\sqrt{x^3+1}}$
(c) $3x^2\sqrt{x^3+1}$
(d) $\frac{3x^2}{2\sqrt{x^3+1}}$
(e) none of the above
5. What is the absolute maximum value of the function $f(x) = -e^{2x}$ on the closed interval $[p, q]$, where $p$ and $q$ are real numbers with $p < q$?

(a) The function does not have an absolute maximum on this interval.
(b) The absolute maximum is $f(p) = -e^{2p}$.
(c) The absolute maximum is the absolute value of $f(p)$, which is $e^{2p}$.
(d) The absolute maximum occurs at $x = q$.
(e) The function has an absolute maximum on this interval, but the absolute max does not occur at an endpoint of the interval.

6. The quotient rule says that the derivative of $f(x)/g(x)$ is given by

(a) $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] + f(x) \frac{d}{dx} [g(x)]}{g^2(x)}$

(b) $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] + f(x) \frac{d}{dx} [g(x)]}{f^2(x)}$

(c) $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{g^2(x)}$

(d) $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x) \frac{d}{dx} [f(x)] - f(x) \frac{d}{dx} [g(x)]}{f^2(x)}$

(e) $\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{\frac{d}{dx} [g(x)] \frac{d}{dx} [f(x)] - \frac{d}{dx} [f(x)] \frac{d}{dx} [g(x)]}{g^2(x)}$
7. \[ \int x^4 + x^2 + x \, dx = \]
(a) \( \frac{1}{3} x^6 + \frac{3}{3} x^3 + \frac{2}{2} x + C \)
(b) \( \frac{x^6}{4} + \frac{x^3}{3} + \frac{x^2}{2} + C \)
(c) \( \frac{x^6}{4} + \frac{x^3}{3} + \frac{x^2}{2} + C_1 \)
(d) \( x^2 + \frac{1}{x} + \frac{1}{x^2} + C \)
(e) \( \frac{x^3}{3} + \ln x - \frac{1}{x} + C \)

8. If \( f(x) = e^{\ln x} \), then \( f^{(2008)}(x) \), the 2008th derivative of \( f(x) \) is
(a) \( e^{\ln x} \)
(b) 1
(c) 0
(d) \( e^{\ln x} \) \( x^{2008} \)
(e) none of the above

9. If the functions \( f \) and \( g \) are continuous and the derivative of \( f(x) \) is \( g(x) \), then
(a) \( \int_1^2 f(x) \, dx = g(2) - g(1) \)
(b) \( \int_1^2 f(x) \, dx = g(1) - g(2) \)
(c) \( \int_1^2 g(x) \, dx = f(2) - f(1) \)
(d) \( \int_1^2 g(x) \, dx = f(1) - f(2) \)
(e) none of the above

NOTE: All the integrals in this question go from 1 to 2, but the function being integrated changes.
10. If we use implicit differentiation to find the slope of the tangent line to the graph of the equation \( x^2 - y^2 = 2x + 4y - 6 \) at the point \((5,3)\), we find that the slope is

(a) \( \frac{3}{7} \)
(b) \( \frac{5}{6} \)
(c) \( 0.8 \)
(d) \( \frac{6}{5} \)
(e) none of the above

11. What is the value of \( \int_0^\infty e^{-x/3} \, dx \)?

(a) 2
(b) 3
(c) 4
(d) the integral diverges
(e) none of the above

12. Finding the derivative of \( f(x) = 3x^2 \) using the definition of derivative forces us to evaluate what limit?

(a) \( \lim_{h \to 0} \frac{(3x+h)^2 - 3x^2}{h} \)
(b) \( \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h} \)
(c) \( \lim_{h \to 0} \frac{(3x+3h)^2 - 3x^2}{h} \)
(d) \( \lim_{h \to 0} \frac{3x^2 + h^2 - 3x^2}{h} \)
(e) \( \lim_{h \to 0} \frac{(x+3h)^2 - 3x^2}{h} \)
13. (20 pts) Evaluate $\int_1^2 x^2 (x^3 + 1)^6 \, dx$
14. (20 pts) Evaluate \[ \int_2^3 x^2 \ln x \, dx \]
15. (30 pts) Find the particular solution of the differential equation \( \frac{dy}{dx} = \frac{2x}{y^2} \) satisfying 
\( y = 3 \) when \( x = 2 \).
16. (30 pts) Find the area of the region enclosed by the line $y = 5x - 6$ and the parabola $y = x^2$. 
17. (30 pts) A closed box with a square base is to have a volume of 12,000 cubic inches. If the material for the top and bottom cost $3 per square inch and the material for the sides costs $2 per square inch, what dimensions will minimize the cost of the box?
18. (25 pts) Let \( f(x, y) = x^3 + 4y^2 - 12x \)

Find the critical points of \( f \) and classify each critical point as relative maximum, relative minimum, or saddle point. Reminder:

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<th>Sign of ( f_{xx} )</th>
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