Final exam on Monday, May 10th 2010, 1:00PM-4:00PM

1. Check that you have all the questions (mentioned in the grade-table alongside) on your exam.

2. This exam is closed book and notes. You may use a scientific calculator but not a graphing calculator or a **cell-phone calculator**.

3. You have 3 hours to do the exam. When the proctor calls time you must stop writing and hand in your papers or you may be given a score of 0%.

4. Do all your work in the spaces provided on these sheets. If you need additional space ask the proctor for more paper and attach it to the exam. Write your name on the paper and clearly identify the problem number.

5. Your solutions are graded primarily for procedure and partial credit is possible. To receive any points you must show the detailed mathematical steps needed to arrive at your answers. A correct answer with no work shown receives little or no credit. Answers must be in simplest form.

1. (10 points) Compute the second-order mixed partial derivative $f_{xy}$ for the function $f(x, y) = x y e^{x y}$.

   a. $x y e^{x y} (1 + x y)$
   b. $y^2 e^{x y} (2 + x y)$
   c. $x^2 e^{x y} (2 + x y)$
   d. $e^{x y} (1 + 3 x y + (x y)^2)$
   e. None of the above

<table>
<thead>
<tr>
<th>Question</th>
<th>Points</th>
<th>Score</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>19</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>21</td>
<td>25</td>
<td></td>
</tr>
<tr>
<td>Total:</td>
<td>300</td>
<td></td>
</tr>
</tbody>
</table>
2. (10 points) What is the Domain of the function \( f(x) = \frac{\sqrt{x - 9}}{x^2 + x - 2} \)

   a. All Real Numbers
   b. All Real Numbers, \( x \neq -2, +1 \)
   c. All Real Numbers, \( x \geq -2 \)
   d. All Real Numbers, \( x \geq 9 \)
   e. None of the above

3. (10 points) Determine the values of \( x \) for which the function \( f(x) = \frac{3x - 2}{x^2 - 3x - 18} \) is NOT Continuous

   a. \( x = -3, +6 \)
   b. \( x = \frac{2}{3}, -3, +6 \)
   c. \( x = -\frac{2}{3}, +3, -6 \)
   d. \( x = +3, -6 \)
   e. None of the above
4. (10 points) Evaluate the limit: \( \lim_{x \to 1} \frac{x^2 + 4x - 5}{x^2 - 1} \)
   
   a. 5
   b. 0
   c. \( \infty \)
   d. 3
   e. None of the above

5. (10 points) Given the function \( f(x) = \frac{1}{3}x^3 - 9x + 2 \) Determine the interval over \( x \), during which the function \( f(x) \) is decreasing.
   
   a. \(-\sqrt{3} < x < +\sqrt{3}\)
   b. \(-3 < x < +3\)
   c. \(-1 < x < +1\)
   d. \( f(x) \) never decreases
   e. None of the above
6. (10 points) Determine, if any, the horizontal asymptotes of \( f(x) = \frac{5x^2}{x^2 - 3x - 4} \)
   a. \( y = 0 \)
   b. \( y = 5 \)
   c. \( y = -4 \)
   d. There isn’t one.
   e. None of the above

7. (10 points) Determine, if any, the vertical asymptotes of \( f(x) = \frac{5x^2}{x^2 - 3x - 4} \)
   a. \( x = -4 \)
   b. \( x = +1, -4 \)
   c. \( x = -1, 4 \)
   d. There aren’t any.
   e. None of the above
8. (10 points) Differentiate the function: $f(x) = e^x \ln x$
   a. $e^x \left[ \frac{1}{x^2} + x \ln x \right]$
   b. $e^x \ln x + x$
   c. $e^x / x$
   d. $e^x \left[ \frac{1}{x} + \ln x \right]$
   e. None of the above

9. (10 points) Determine the anti-derivative for: $\int x(2x + 1)^2 \, dx$
   a. $\frac{2}{3} (2x + 1)^3 + C$
   b. $x^4 + \frac{4}{3} x^3 + \frac{1}{2} x^2 + C$
   c. $\frac{x^2}{2} \left( \frac{4}{3} x^3 + \frac{4}{3} x^2 + x \right) + C$
   d. $\frac{x^2}{2} (2x + 1)^3 + C$
   e. None of the above
10. (10 points) Evaluate $\int_1^6 x^2(x - 1)dx$
   a. $\frac{3025}{12}$
   b. $\frac{3024}{12}$
   c. $\frac{3023}{12}$
   d. 252
   e. None of the above

11. (10 points) Using Integration by Parts, Integrate: $\int \frac{\ln x}{x^2} dx$
   a. $\frac{1}{x}[\ln x - 1] + C$
   b. $\frac{1}{x}[\ln x + 1] + C$
   c. $-\frac{1}{x}[\ln x - 1] + C$
   d. $-\frac{1}{x}[\ln x + 1] + C$
   e. None of the above
12. (10 points) Determine the particular solution for: \( \frac{dy}{dx} = e^{5x} \) where \( y = 1 \) when \( x = 0 \)

   a. \( \frac{1}{5}e^{5x+1} + \frac{6}{5} \)
   b. \( \frac{1}{5}e^{5x+5} \)
   c. \( \frac{1}{5}e^{5x} + \frac{4}{5} \)
   d. \( \frac{1}{5}e^{5x} \)
   e. None of the above

13. (10 points) Evaluate the integral: \( I = \int_{0}^{\infty} e^{-x} \, dx \)

   a. 1
   b. -1
   c. 0
   d. \( \infty \)
   e. None of the above
14. (10 points) Integrate the following function: \( \int x(x^2 + 1)^3 \, dx \)

\[
\begin{align*}
a. & \quad \frac{1}{6} u^6 + C \\
b. & \quad \frac{x^{16}}{16} + \frac{x^7}{7} + C \\
c. & \quad \frac{1}{6} (x^2 + 1)^6 + C \\
d. & \quad \frac{1}{12} (x^2 + 1)^6 + C \\
e. & \quad None \ of \ the \ above
\end{align*}
\]

15. (10 points) Find the probability \( P(1 \leq x \leq 2) \) for the probability density function

\[
f(x) = \begin{cases} 
    k(3 - x) & \text{if } 0 \leq x \leq 3 \\
    0 & \text{else.}
\end{cases}
\]

\[
\begin{align*}
a. & \quad 1/3 \\
b. & \quad 1/9 \\
c. & \quad 2/9 \\
d. & \quad 2/3 \\
e. & \quad None \ of \ the \ above
\end{align*}
\]
16. (25 points) Use the algebraic definition (difference quotient) of a derivative to determine the derivative $f'(x)$ of the function $f(x) = \frac{1}{x + 2}$. 
17. (25 points) Find the equation of the tangent line at $x = 1/4$, for the following function.

\[
\frac{1}{x} - \frac{1}{y} = 2
\]
18. (25 points) Sketch the graph of the function $f(x) = e^{-x^2}$ showing the critical points, the points of inflection, and the asymptotes. State the intervals over $x$ where $f(x)$ is increasing and decreasing; and the intervals over $x$ where $f(x)$ is concave up or down.
19. (25 points) Determine the area of the region between the curves \( y = x^3 \) and the line \( y = 9x \), for \( x \geq 0 \).
20. (25 points) Find the general solution of the differential equation: \( \frac{dy}{dx} = x^5 e^{x^3} \)
21. (25 points) Find the critical points and classify each as a relative maximum, relative minimum or a saddle point for the function \( f(x, y) = x^2 + y^3 + 6xy - 7x - 6y \).