MATH 1151
Spring 2003
FINAL EXAM

Name: __________________________ Signature __________________________

TA: ___________________ Discussion: ___________________ ID: __________

READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 15 pages, including the cover page. Check to see if any are missing. PRINT
all requested information above and sign your name. Put your initials on the top of every page,
in case the pages become separated. A scientific calculator may be used, but Textbooks and
notes are not permissible. Do your work in the blank spaces and in the back of pages of this
booklet. Show all your work.

This exam consists of 12 machine-graded questions, worth 8 points each, and 8 hand-graded
questions, worth 14, 12 or 10 points, making a total of 200 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1–12):
You MUST use a soft pencil (No.1 or No.2) to answer this part. Do not fold or tear the
answer sheet, and carefully enter all the requested information according to the instructions
that you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER
SHEET. When you have decided on a correct answer to a given question, circle the answer in
this booklet and blacken completely the corresponding circle in the answer sheet. If you erase
something, do so completely. Each question has a correct answer. If you give two different
answers, the question will be marked wrong. There is no penalty for guessing, but if you don’t
answer a question, skip the corresponding line in the answer sheet. Go on to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 13–20):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the School
of Mathematics may for any reason request a regrade of the machine graded part. All regrades
will be based on responses in the test booklet, and not on the machine graded response sheet.
Therefore work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM, place the answer sheet between
two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE
ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning in your
exam.

Hand-graded part: __________________________ Machine-graded part: __________________________

Total __________________________


Please leave these boxes blank!
[1] Find the exact value of the expression

\[\sin(77^\circ) \cos(47^\circ) - \cos(77^\circ) \sin(47^\circ)\]

a) \(\frac{1}{2}\)
b) \(-\frac{1}{2}\)
c) 0
d) \(\frac{\sqrt{3}}{2}\)
e) \(-\frac{\sqrt{3}}{2}\)

[2] Find the exact value of the expression

\[\sin \left( \cos^{-1} \left( \frac{5}{13} \right) - \cos^{-1} \left( \frac{4}{5} \right) \right)\]

a) \(\frac{63}{65}\)
b) \(\frac{20}{65}\)
c) 0
d) \(\frac{33}{65}\)
e) \(\frac{77}{65}\)
[3] The polar form of \( z = (1 - i)^3 \) is:

a) \( \sqrt{2}(\cos \frac{\pi}{4} - i \sin \frac{\pi}{4}) \)

b) \( 2\sqrt{2}(\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) \)

c) \( 2\sqrt{2}(\cos \frac{5\pi}{4} - i \sin \frac{5\pi}{4}) \)

d) \( \sqrt{8}(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4}) \)

e) None of the above

[4] What is the correct value of the sum

\[
\sum_{k=1}^{12} \left( \frac{3}{2} k^3 - 1 \right)
\]

a) 9824

b) 1342

c) 9114

d) 9672

e) None of the above
What is the sum equal to?
\[ \sum_{k=1}^{11} 2(-3)^{k-1} \]

a) None of the following
b) \[ 2 \cdot \frac{1-3^{11}}{1-(-3)} \]
c) \[ 2 \cdot \frac{1-(3)^{11}}{1-(-3)} \]
d) \[ 2 \cdot \frac{1-3^{11}}{1-3} \]
e) \[ -2 \cdot \frac{1-3^{11}}{1-3} \]

What is the equation of a ellipse with foci at \((-4, 2)\) and \((-4, 8)\) and vertex at \((-4, 10)\)? (It may help to draw a picture!)
a) \[ \frac{(x-4)^2}{16} + \frac{(y-5)^2}{25} = 1 \]
b) \[ \frac{(x+4)^2}{16} + \frac{(y-5)^2}{25} = 1 \]
c) \[ \frac{(x-4)^2}{25} + \frac{(y-5)^2}{16} = 1 \]
d) \[ \frac{(x+4)^2}{25} + \frac{(y-5)^2}{16} = 1 \]
e) \[ \frac{(x-4)^2}{25} - \frac{(y-5)^2}{16} = 1 \]
[7] What is the area of a triangle which has 2 adjacent sides equal to 5 and 4 respectively, and the angle formed by these sides is 30°?

a) 5
b) 10
c) $10\sqrt{3}$
d) $5\sqrt{3}$
e) None of the above

[8] Determine which of the four sequences below are arithmetic.

I) 4, 12, 36, 108, 972, ...

II) 2, 4, 6, 10, 12, ...

III) 2, −3, −8, −13, −18, ...

IV) 2, −1, −4, −7, −10, ...

a) II, III and IV only
b) III and IV only
c) II and III only
d) I only
e) None of them
[9] What is the sum equal to?

\[ \sum_{k=1}^{\infty} 2 \left( -\frac{1}{3} \right)^{k-1} \]

a) 1.5
b) Does not exist
c) \(-0.6667\)
d) \(-1\)
e) 3

[10] Consider the equation \(x^2 = 4y\). In polar coordinates, this equation simplifies to?

a) \( r \sin^2 \theta = 4 \cos \theta \)
b) \( 4 \cos^2 \theta = r \sin \theta \)
c) \( 4 \sin^2 \theta = r \cos \theta \)
d) \( r \cos^2 \theta = 4 \sin \theta \)
e) None of the above
[11] Find a third-degree polynomial function with real coefficients and with zeros 1 and $3 + i$.

   a) $x^3 - 7x^2 + 4x - 10$
   b) $x^3 - 5x^2 + 4x + 10$
   c) $x^3 + 7x^2 + 4x - 10$
   d) $x^3 + 7x^2 + 16x + 10$
   e) $x^3 - 7x^2 + 16x - 10$

[12] What the period of the function:

   \[ f(x) = -\frac{1}{2} \sin(4x + 3\pi) \]

   a) 4
   b) $\pi$
   c) $\frac{\pi}{2}$
   d) $2\pi$
   e) $\frac{3}{4}\pi$
[13] Solve the following trigonometric equation for $0 \leq \theta < 2\pi$:

$$1 + \sqrt{3} \cos \theta + \cos(2\theta) = 0$$
14. Find the coordinates of the center and vertices of the conic section and draw them together with the curve. Your graph need not be perfect, but you should use the best techniques shown in class to make it as accurate as possible.

\[-4x^2 + y^2 - 16x - 2y - 19 = 0\]
[15] Given the polynomial \( f(x) = x^4 - 8x^3 + 16x^2 + 8x - 17. \)

a) (6 points) Write \( f(x) \) in factored form.
b) (8 points) Find all of the complex zeros.
Graph the following system of inequalities.

\[
\begin{align*}
    x & \geq 0 \\
    y & \geq 0 \\
    x^2 + y^2 & \leq 4 \\
    (x - 1)^2 + y^2 & \leq 4
\end{align*}
\]
[17] Prove the trigonometric identity:

\[ 1 - 8 \sin^2(\theta) \cos^2(\theta) = \cos(4\theta) \]
In the triangle below in addition to the angles mentioned in the picture we know that $AB = 3$, $CD = \frac{1}{4}$, $CE = \frac{1}{4}$. Find the length $AD + DE + EB$. (The picture might not be perfectly according to the data. Use the given data in the problem.)
Given the complex number $z = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} \cdot i$.

a) (4 points) Find the polar form of $z$

b) (4 points) Find $z^3$

c) (6 points) Find all complex 3 roots of $z$.

Give all the answers in polar form.
[20] If \( \sin \alpha = \frac{5}{6} \) and \( \cos \beta = \frac{2}{5} \), where \( \frac{\pi}{2} \leq \alpha \leq \pi \) and \( -\frac{\pi}{2} \leq \beta \leq 0 \), find the exact value of \( \cos(\alpha + \beta) \).