READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 17 pages including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. TURN OFF CELL PHONES! Scientific calculators allowed, but not graphing calculators. One crib sheet 8.5” x 11” is allowed, written in your own handwriting and only on one side. NO OTHER TEXTBOOKS OR NOTES ARE PERMITTED! Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 10 multiple choice problems each worth 15 points, and a hand-graded part with 6 problems counting 25 points each, for a total score of 300 points. You have 3 hours to work on this exam.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-10):
You MUST use a soft pencil (No. 1 or No 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing, but if you don’t answer a question, skip the corresponding line in the answer sheet. Go on to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 11-16):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. Therefore work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning in your exam.

Multiple choice part ______ Hand-graded part _______

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Multiple Choice Part

1. An equation for the tangent plane to the surface $x^2 - x + 2y^2 + e^z + z = 3$ at the point $(1, 1, 0)$ is

   A. $z = x + 4y - 5$
   B. $x + 4y + 2z = 5$
   C. $x + 4y - 2z = 5$
   D. $x + 4y + 2z = 0$
   E. $(2x - 1)(x - 1) + 4y(y - 1) + (e^z + 1)z = 0$
2. If \( u = x^4 + 2y^3 \) and \( x = e^s \) and \( y = e^{s-t} \) then \( \frac{\partial u}{\partial t} \) at \( s = 2, \ t = 1 \) equals

A. \( e^8 + 2e^3 \)

B. \( 8e^6 - 6e^3 \)

C. \( 4e^8 - 6e^3 \)

D. \( 8e^8 - 6e^3 \)

E. \( 8e^8 + 6e^3 \)
3. Let \( f(x, y) = x^2 - 2y^3 - 6xy - 18y + 2x + 14 \). Then

A. \( f \) has saddle points at both \((2, 1)\) and \((5, 2)\), and no other critical points.

B. \( f \) has a local minimum at \((-4, -1)\) and a local maximum at \((-7, -2)\) and no other critical points.

C. \( f \) has a saddle point at \((-4, -1)\) and a local minimum at \((-7, -2)\), and no other critical points.

D. \( f \) has a local maximum at \((-4, -1)\) and a local minimum at \((-7, -2)\), and no other critical points.

E. \( f \) has a saddle point at \((-4, -1)\) and a local maximum at \((-7, -2)\), and no other critical points.
4. Evaluate the integral
\[ \int_0^4 \int_{y^2}^{16} y \sin(x^2) \, dx \, dy \]

A. \(-2 \cos 256\)

B. \(2(1 - \cos 256)\)

C. \(\frac{1}{4} \cos 256\)

D. \(\frac{1}{4}(\cos 256 - 1)\)

E. \(\frac{1}{4}(1 - \cos 256)\)
5. Let $D$ be the circular region $\{x^2 + y^2 \leq 4x\}$. Then, in terms of polar coordinates,

$$\int \int_D (x^2 + y^2)^{5/2} \, dx \, dy$$

equals

A. $\int_{-\pi/2}^{\pi/2} \int_0^4 r^5 \, dr \, d\theta$

B. $\int_{-\pi/2}^{\pi/2} \int_0^4 r^6 \, dr \, d\theta$

C. $\int_0^{2\pi} \int_0^4 r^6 \, dr \, d\theta$

D. $\int_0^{2\pi} \int_0^{4 \cos \theta} r^6 \, dr \, d\theta$

E. $\int_{-\pi/2}^{\pi/2} \int_0^{4 \cos \theta} r^6 \, dr \, d\theta$
6. Let \( C \) be the closed curve in \( \mathbb{R}^2 \) consisting of the line segment on the \( x \)-axis joining \((2, 0)\) to \((-2, 0)\), and of the semicircle \( x^2 + y^2 = 4 \), \((y < 0)\) (also joining \((2, 0)\) and \((-2, 0)\)), oriented counterclockwise. Then

\[
\int_C (x^2 + 8y) \, dx + (3x + 4y + x^2) \, dy
\]

equals

A. \(-10\pi\)
B. \(10\pi\)
C. \(-20\pi\)
D. \(22\pi\)
E. none of the above.
7. Find a \( \phi \) whose gradient is the vector field \( \langle 24x^2 - 5y^2 - 6xz^2, -10xy + 8z^3, -6x^2z + 24yz^2 \rangle \).

A. \( \langle 8x^3 - 5xy^2 - 3x^2z^2, -5xy^2 + 8yz^3, -3x^2z^2 + 8yz^3 \rangle \)

B. \( 8x^3 - 10xy^2 - 6x^2z^2 + 16yz^3 \)

C. \( \langle 48x - 6z^2, -10x, -6x^2 + 48yz \rangle \)

D. \( 38x - 6x^2 - 6z^2 + 48yz \)

E. \( 8x^3 - 5xy^2 - 3x^2z^2 + 8yz^3 \).
8. Let $C$ be the curve in $\mathbb{R}^2$ which goes from the point $(9, 3)$ to $(1, 1)$ along the parabola $y = \sqrt{x}$ and then continues on to $(0, 0)$ along the line $y = x$. Then the integral

$$\int_C (y^2 + 1)dx + (2xy - 2)dy$$

equals

A. 84
B. −84
C. 165
D. −165
E. 0
9. The domain of integration for the integral
\[ \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{4-x^2-y^2}}^{\sqrt{8-x^2-y^2}} f(x,y,z) \, dz \, dy \, dx \]
is

A. a subset of \( \mathbb{R}^3 \) bounded from below by the \( xy \)-plane, on the sides by a cylinder, and from above by a paraboloid.

B. a subset of \( \mathbb{R}^3 \) bounded from below by a cone and from above by a paraboloid.

C. a subset of \( \mathbb{R}^3 \) bounded from below by a cone and from above by a sphere.

D. a subset of \( \mathbb{R}^3 \) bounded from below by a cone and from above by a hyperboloid.

E. a subset of \( \mathbb{R}^3 \) bounded from below by a sphere and from above by a paraboloid.
10. The surface area of the part of the surface $z = xy$ situated inside the cylinder $x^2 + y^2 = 8$ equals

A. $4\pi$

B. $\frac{52}{3}\pi$

C. $18\pi$

D. $\frac{26}{3}\pi$

E. $\frac{32\pi \sqrt{2}}{3}$
Hand-Graded Part

11. (25 pts) The temperature at a point \((x, y, z)\) is given by

\[ T(x, y, z) = 3xe^y + ye^z + ze^x. \]

(a) Find the rate of change of the temperature at the point \(P(0, 0, 0)\) in the direction toward the point \((5, 1, -2)\).

(b) Starting at the point \(P\), in which direction does the temperature increase the fastest? (Please give answer as a unit vector.)

(c) What is the maximum rate of increase at \(P\)?
12. (25 pts) Find the minimum and maximum values of the function $f(x, y, z) = xyz$ subject to the constraint $x^2 + 3y^2 + 4z^2 = 12$. Find a point where the maximum occurs, and a point where the minimum occurs.
13. (25 pts) Let

\[ \vec{F}(x, y, z) = (\sin z + xy^2)\vec{i} + x^2e^{3z}\vec{j} + (\cos^3 x + x^2z)\vec{k}. \]

Let \( T \) be the surface bounding the region \( R \) given by \( x^2 + y^2 \leq z \leq 6 - \sqrt{x^2 + y^2} \), oriented outward. Use the divergence theorem to evaluate the flux

\[ \iint_T \vec{F} \cdot d\vec{S}. \]
14. (25 pts) (a) Use spherical coordinates to evaluate the triple integral

\[ \iiint_E \cos^2\left(\frac{(x^2 + y^2 + z^2)^{3/2}}{2}\right) \, dV, \]

where \( E \) is the region enclosed by the sphere \( x^2 + y^2 + z^2 = 4 \) in the first octant \( \{x \geq 0, \ y \geq 0, \ z \geq 0\} \).

(b) What is the answer if \( x^2 + y^2 + z^2 \) is replaced by \( x^2 + 5y^2 + 3z^2 \) in both the integral and the region?
15. (25 pts) Consider the portion $P$ of the sphere $x^2 + y^2 + z^2 = 1$ located in the first octant \( \{ x \geq 0, y \geq 0, z \geq 0 \} \). Calculate the work done by the force field

$$\vec{F}(x, y, z) = xe^x \hat{i} + (x^4 + ye^y) \hat{j} + (2y + z \sin^3 z) \hat{k}$$

as a particle moves under the field's influence along the edge of $P$ once in a counterclockwise direction (as seen from very high above the $xy$-plane).
16. (25 pts) Let $T$ be the portion of the surface $x^2 = y^2 + z^2$ lying between the planes $x = 0$ and $x = 2$ and above the plane $z = 0$. Calculate the surface integral

$$\iint_T (2 + x^2 y^2) \, dS$$

(i.e. the mass of surface $T$ if its density is $2 + x^2 y^2$).