READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 12 pages including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. TURN OFF CELL PHONES! Scientific calculators allowed, but not graphing calculators. NO TEXTBOOKS OR NOTES ARE PERMITTED! Do your work in the blank spaces and back of pages of this booklet. Show all your work.

There are 10 multiple choice problems each worth 15 points, and a hand-graded part with 6 problems counting 25 points each, for a total score of 300 points. You have 3 hours to work on this exam.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-10):
You MUST use a soft pencil (No. 1 or No 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing, but if you don’t answer a question, skip the corresponding line in the answer sheet. Go on to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 11-16):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. Therefore work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning in your exam.

Multiple choice part _______ Hand-graded part _______

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Total _______

Letter Grade _______
Multiple Choice Part

1. An equation for the tangent plane to the surface $xz^2 - 2y^3 + 1 = 0$ at the point $(1, 1, 1)$ is
   
   A. $z^2(x - 1) - 6y^2(y - 1) + 2xz(z - 1) = 0$
   B. $z - 1 = (x - 1) - 6(y - 1)$
   C. $x - 6y + 2z = -3$
   D. $x - 6y + 2z = -1$
   E. $(x - 1) + 6(y - 1) + 2(z - 1) = 0$

2. If $u = x^5 + 2y^3$ and $x = e^{st}$ and $y = e^{s-t}$ then $\frac{\partial u}{\partial t}$ at $s = 2$, $t = 1$ equals
   
   A. $e^{10} + 2e^3$
   B. $10e^{10} - 6e^3$
   C. $10e^{10} + 6e^3$
   D. $10e^8 - 6e^3$
   E. $5e^{10} - 6e^3$
3. Let \( f(x, y) = x^2 + 2y^3 - 6xy + 18y - 2x + 16 \). Then

A. \( f \) has saddle points at both \((-2, -1)\) and \((-5, -2)\), and no other critical points.

B. \( f \) has a local maximum at \((4, 1)\) and a local minimum at \((7, 2)\) and no other critical points.

C. \( f \) has a saddle point at \((4, 1)\) and a local minimum at \((7, 2)\), and no other critical points.

D. \( f \) has a saddle point at \((4, 1)\) and a local maximum at \((7, 2)\), and no other critical points.

E. \( f \) has a local minimum at \((4, 1)\) and a local maximum at \((7, 2)\), and no other critical points.

4. Changing the order of integration in the iterated integral

\[
\int_0^2 \int_{\frac{x^2}{2}}^{\frac{8}{y-1}} f(x, y)dydx
\]

leads to

(A) \( \int_0^5 \int_{\frac{\sqrt{8}}{y-1}} f(x, y)dx dy \)

(B) \( \int_0^2 \int_{\frac{\sqrt{8}}{y-1}} f(x, y)dx dy \)

(C) \( \int_0^1 \int_{0}^{2y} f(x, y)dx dy + \int_1^5 \int_{\frac{\sqrt{8}}{y-1}} f(x, y)dx dy \)

(D) \( \int_0^1 \int_{0}^{2y} f(x, y)dx dy + \int_1^5 \int_{\frac{\sqrt{8}}{y-1}} f(x, y)dx dy \)

(E) none of the above.
5. Let $D$ be the circular region $\{x^2 + y^2 \leq 6x\}$. Then, in terms of polar coordinates,

$$\int \int_D \sqrt{x^2 + y^2} \, dx \, dy$$

equals

A. $\int_{-\pi/2}^{\pi/2} \int_0^6 r \, dr \, d\theta$

B. $\int_{-\pi/2}^{\pi/2} \int_0^6 r^2 \, dr \, d\theta$

C. $\int_0^{2\pi} \int_0^6 r^2 \, dr \, d\theta$

D. $\int_0^{2\pi} \int_0^6 \cos \theta \, r^2 \, dr \, d\theta$

E. $\int_{-\pi/2}^{\pi/2} \int_0^6 \cos \theta \, r^2 \, dr \, d\theta$

6. Consider the region bounded by the lines $y = 1$ and $y = 3$, the hyperbola $xy = 3$, and the $y$-axis. The centroid of this region (i.e., the center-of-mass of this region for density $= 1$) is the point

(A) $(\ln 3, \frac{1}{2} \ln 3)$

(B) $\left(\frac{1}{\ln 3}, \frac{2}{\ln 3}\right)$

(C) $\left(\frac{2}{\ln 3}, \frac{1}{\ln 3}\right)$

(D) $(\frac{3}{2}, 2)$

(E) $(2, 2)$
7. Let \( C \) be the closed curve in \( \mathbb{R}^2 \) consisting of the line segment from \((4, 0)\) to \((-4, 0)\), and the semicircle \( x^2 + y^2 = 16 \) \((y < 0)\). Orient \( C \) counter-clockwise. Then the integral

\[
\int_C (6x^2 + 7y)dx + (5x + 5y + 16)dy
\]

equals

(A) \(40\pi\)

(B) \(96\pi\)

(C) \(16\pi\)

(D) \(-16\pi\)

(E) 0

8. Let \( \vec{F}(x, y, z) = 3x^2y\vec{i} + (-yz)\vec{j} + 2xz\vec{k} \). Then \( \text{curl}\vec{F} \) is

(A) \(6xy\vec{i} - z\vec{j} + 2x\vec{k}\)

(B) \(-y\vec{i} + 2z\vec{j} + 3x^2\vec{k}\)

(C) \(-y\vec{i} + 2z\vec{j} - 3x^2\vec{k}\)

(D) \(y\vec{i} + 2z\vec{j} - 3x^2\vec{k}\)

(E) \(y\vec{i} - 2z\vec{j} - 3x^2\vec{k}\)
9. Let $C$ be the path that goes from $(-3, 0)$ to $(3, 0)$ along the semicircular arc $y = \sqrt{9 - x^2}$, then to $(-3, 3)$ along the line $x + 2y = 3$. Then

$$\int_C (2xy + 3)\,dx + (x^2 + 1)\,dy$$

equals
(A) 57
(B) -57
(C) 0
(D) 30
(E) none of the above.

10. Let $B$ be the solid in $\mathbb{R}^3$ bounded from below by the elliptic paraboloid $z = x^2 + 2y^2$ and from above by the parabolic cylinder $z = 10 - x^2$. Then the integral $\iiint_D f(x, y, z)\,dV$ can be evaluated as the following iterated integral:

(A) $\int_{-\sqrt{10}}^{\sqrt{10}} \int_{-\sqrt{5}}^{\sqrt{5}} \int_{x^2+2y^2}^{10-x^2} f(x, y, z)\,dz\,dy\,dx$

(B) $\int_{-\sqrt{10}}^{\sqrt{10}} \int_{-\sqrt{5-x^2/2}}^{\sqrt{5-x^2/2}} \int_{x^2+2y^2}^{10-x^2} f(x, y, z)\,dz\,dy\,dx$

(C) $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} \int_{0}^{10} f(x, y, z)\,dz\,dy\,dx$

(D) $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} \int_{\sqrt{x^2+2y^2}}^{\sqrt{10-x^2}} f(x, y, z)\,dz\,dy\,dx$

(E) $\int_{-\sqrt{5}}^{\sqrt{5}} \int_{-\sqrt{5-x^2}}^{\sqrt{5-x^2}} \int_{x^2+2y^2}^{10-x^2} f(x, y, z)\,dz\,dy\,dx$
11. (25 pts) The temperature at a point \((x, y, z)\) is given by

\[ T(x, y, z) = x^3 + 4xy + xyz^2. \]

(a) Find the rate of change of the temperature at the point \(P(1, 1, -1)\) in the direction toward the point \(Q(2, 3, 3)\).

(b) Starting at the point \(P\), in which direction does the temperature *increase* the fastest? (Please give answer as a unit vector.)

(c) What is the maximum rate of increase at \(P\)?
12. (25 pts) Find the point or points on the surface $z^2 = xy + 64$ which lie closest to the origin. (For full credit, you must justify why your answer gives the absolute minimum.)
13. (25 pts) Let $E$ be the region in $xyz$-space which lies above the $xy$-plane, outside the sphere $x^2 + y^2 + z^2 = 1$, and inside the sphere $x^2 + y^2 + z^2 = 9$. If the density at each point of $E$ is the distance of that point to the origin, find the mass and center-of-mass of $E$. 
14. (21+4 pts) (a) Calculate the outward flux of the vector field

$$\vec{F}(x, y, z) = < 2x^3, 5xz^2, x^2 + 6y^2z >$$

across $S$ which is the surface of the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the plane $z = 0$.

(b) What is the outward flux through just the paraboloid? (Hint: consider the flux through the bottom piece first.)
15. (25 pts) Find the surface area of the portion of the paraboloid \( z = x^2 - 3y^2 + 2 \) that lies inside the cylinder \( x^2 + 9y^2 = 1 \).
16. (25 pts) Calculate the work done by the force field

\[ \vec{F}(x, y, z) = \langle e^{4x} + z^2, e^{4y} + x^2, e^{4z} + y^2 \rangle \]

when a particle moves under its influence around the edge of the part of the sphere \( x^2 + y^2 + z^2 = 9 \) that lies in the first octant, in a counterclockwise direction as viewed from above (i.e., from \( z = +\infty \)).