READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 18 pages including this cover page. Check to see if any are missing.
PRINT on the upper right-hand corner all the requested information, and sign your name.
Put your initials on the top of every page, in case the pages become separated. Textbooks,
notes are not permitted. Do your work in the blank spaces and back of pages of this
booklet. Show all your work. This is a closed book exam.

There are 10 multiple choice problems each worth 15 points, and a hand-graded part with
5 problems with variable points, for a total score of 300 points. You have 3 hours to work
on this exam.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-10):
You MUST use a soft pencil (No. 1 or No 2) to answer this part. Do not fold or tear the
answer sheet, and carefully enter all the requested information according to the instruc-
tions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER
SHEET. When you have decided on a correct answer to a given question, circle the answer
in this booklet and blacken completely the corresponding circle in the answer sheet. If you
erase something, do so completely. Each question has a correct answer. If you give two
different answers, the question will be marked wrong. There is no penalty for guessing,
but if you don’t answer a question, skip the corresponding line in the answer sheet. Go on
to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 11-15):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the
School of Mathematics may for any reason request a regrade of the machine graded part.
All regrades will be based on responses in the test booklet, and not on the machine graded
response sheet. Any problem for which the answer is not indicated in the test booklet,
or which has no relevant accompanying calculations will be marked wrong on the regrade.
Therefore work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet
between two pages of this booklet (make a sandwich), with the side marked “GENERAL
PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when
turning in your exam.

Multiple choice part ______ Hand-graded part ______

Total ______

Letter Grade ______

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Multiple-Choice Problems

1. $z - xy + x^2 = 0$ is a surface; its tangent plane at $(1, -1, -2)$ is
   a. $3x - y + z = 2$
   b. $x - y + z = 0$
   c. $x - 2y - z = 5$
   d. $3x - y - z = 6$
   e. $2x - 2y - z = 6$
2. $f$ and $g$ are functions of $x$ and $y$. $f(1, 2) = 2$ and $g(1, 2) = 1$. Also at $(1, 2)$

$$df = -2dx + 5dy$$
$$dg = 4dx - 3dy$$

$z$ is a function of $(x, y)$ defined implicitly by $z^2f + zg + x - y = 0$ with $z(1, 2) = -1$. Then

$$\frac{\partial z}{\partial x}(1, 2) =$$

a) $1/2$

b) $-1/2$

c) $5/3$

d) $-5/3$

e) $-2$
3. \( f(x, y) = -x^2 + xy + y^2 + 3x + y \)
   
a) \( f \) has no critical point.
   
b) \( f \) has one critical point and it is a local max.
   
c) \( f \) has one critical point and it is a local min.
   
d) \( f \) has one critical point and it is a saddle point.
   
e) \( f \) has two critical points.
4. $x = uv, \quad y = u^2$ gives a transformation from the $(u,v)$-plane to the $(x,y)$-plane. It maps the square $0 \leq u \leq 1, \ 0 \leq v \leq 1$ onto a domain $\Omega$.

$$\int \int_{\Omega} xy \, dx \, dy$$

equals

a) 1  
b) 1/6  
c) 2  
d) 1/3  
e) -1/6
5. A plane lamina in polar coordinates is given by \( 0 \leq r \leq \sin \theta, \)
\( 0 \leq \theta \leq \pi/2 \). It has mass per unit area = \( r \). The total mass equals

a) \( \frac{1}{2} \)

b) \( \frac{1}{3} \)

c) \( \frac{3}{4} \)

d) \( \frac{2}{3} \)

e) \( \frac{2}{9} \)
6. Let \( C \) be the triangle with vertices

\((-1,0)\) \( (0,2) \) \( (1,0) \)

oriented counterclockwise. Let

\[ I = \int_C \left( \sin(x + y) - e^x \right) dx + (\sin(x + y) + xy) dy. \]

\( I \) equals

a) \( \frac{1}{2} \)

b) \( 2 \)

c) \( -2 \)

d) \( \frac{5}{4} \)

e) \( \frac{4}{3} \)
7. \( \vec{F} = <3x^2y + y^2 + z^2, x^3 + 2xy, z^2 + 2xz> \) is a force field on \( \mathbb{R}^3 \).

a) \( \vec{F} \) is not conservative.

\( \vec{F} \) is conservative and the work done on a particle moving from \((x, y, z)\) to the origin is

b) \( x^3y + x(y^2 + z^2) - \frac{1}{3}z^3 \)

c) \( x^3y - x(y^2 + z^2) - \frac{1}{3}z^3 \)

d) \( x^3y + x(y^2 + z^2) + \frac{1}{3}z^3 \)

e) \( -x^3y - x(y^2 + z^2) - \frac{1}{3}z^3 \)
8. A material of mass density $= 8$ fills the region below the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and above the region in the $(x, y)$-plane where $0 \leq r \leq \theta \leq 1$. Its moment $M_{xy}$ equals

a) $\frac{1}{5}$

b) $\frac{2}{3}$

c) $\frac{7}{15}$

d) $\frac{11}{15}$

e) $\frac{1}{4}$
9. The area of the part of the surface $z = xy$ inside the cylinder $x^2 + y^2 = 8$ equals
   a) $4\pi$
   b) $20\pi$
   c) $\frac{52}{3}\pi$
   d) $\frac{50}{3}\pi$
   e) $\frac{26}{3}\pi$
10. The surfaces \( z = x^2 + 2y^2 \) and \( x + y + 2z = 11 \) intersect at \((-2, 1, 6)\). The tangent line at this point to the curve of intersection is

\[
\begin{align*}
x &= -2 + 7t \\
a) \quad y &= 1 + 7t \\
\quad z &= 6 - 8t \\
\quad x &= -2 + 9t \\
\quad y &= 1 + 7t \\
\quad z &= 6 - 8t \\
\quad x &= -2 - 4t \\
b) \quad y &= 1 + 4t \\
\quad z &= 6 - t \\
\quad x &= -2 + t \\
c) \quad y &= 1 + t \\
\quad z &= 6 + 2t \\
\quad x &= -2 + 2t \\
d) \quad y &= 1 - 2t \\
\quad z &= 6 + t \\
\quad e) \quad y &= 1 + 7t \\
\quad z &= 6 - 8t \\
\quad x &= -2 + 9t \\
e) \quad y &= 1 + 7t \\
\quad z &= 6 - 8t \\
\quad x &= -2 - 4t
\end{align*}
\]
SHOW ALL YOUR WORK

11. (20 points) Let $f(x, y) = 1 + xy - x - 2y$. Let $R$ be the closed triangular region with vertices $(1, 0)$, $(5, 0)$, and $(0, 3)$. Find the absolute min and max of $f$ on $R$. 
12. (20 points) Use Lagrange multipliers to find the maximum value of the function $f(x, y) = x^2y$ subject to the constraint $x^2 + 2y^2 = 6$. 
13. Let:

\[ F = xyi + \cos(y)j; \]

\( C_1 \) be the path from \((0, 0)\) to \((1, 1)\) along the parabola \( y = x^2; \)

\( C_2 \) be the path from \((0, 0)\) to \((1, 1)\) along the line \( y = x. \)

\( D \) be the region between the line \( y = x \) and \( y = x^2. \)

(a) (15 points) Evaluate \( \int_{C_1} F \cdot dr. \)

(b) (15 points) Evaluate \( \int_{C_2} F \cdot dr. \)
(c) (10 points) Use Green's Theorem, and your answers to (a) and (b) to evaluate
\[ \iint_D x \, dA. \]
14. (20 points) Let

\[ \mathbf{F}(x, y, z) = x^2 \mathbf{i} + xy^2 \mathbf{j} + z^4 \cos(z) \mathbf{k} \, . \]

Let \( C \) be the curve that is the intersection of the plane \( x + y + z = 3 \) and the cylinder \( x^2 + y^2 = 1 \). Assume that \( C \) is oriented counterclockwise when viewed from above. Evaluate \( \int_C \mathbf{F} \cdot d\mathbf{r} \). (Hint: If you are clever, this is quite easy!)
15. Let:

$E$ be the solid that is bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$;
$S$ be the boundary of $E$;
$S_1$ be the top boundary of $E$;
$S_2$ be the side boundary of $E$.
(So $S = S_1 \cup S_2$).

Assume that each surface $S_1$ and $S_2$ is oriented so that the normal vector at each point
on the surface points outward from the solid $E$. Let

$$\mathbf{F}(x, y, z) = (\cos z + xy^2)\mathbf{i} + xe^{-x}\mathbf{j} + x^2z\mathbf{k}.$$  

(a) (20 points) Use the Divergence Theorem to evaluate the surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S}.$$
(b) (20 points) Evaluate
\[ \int \int_{S_1} \mathbf{F} \cdot d\mathbf{S} . \]

(c) (10 points) Evaluate
\[ \int \int_{S_2} \mathbf{F} \cdot d\mathbf{S} . \]

(This is easy.) Explain how you get your answer.