(1) This exam contains 15 pages (including this cover page) and 20 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

(2) No textbooks or notes will be allowed in this exam. Scientific calculators may be used, but not graphing calculators. Please turn off cell phones.

(3) Do not give numerical approximations to quantities such as \( \sin 5, \pi, \text{ or } \sqrt{2} \). However, you should simplify \( \cos \frac{\pi}{2} = 0, e^0 = 1 \), and so on.

(4) For the problems 13–20, show your work, in a reasonably neat and coherent way, in the space provided. All answers must be justified by valid mathematical reasoning. To receive full credit on a problem, you must show enough work so that your solution can be followed by someone without a calculator.

(5) Mysterious or unsupported answers will not receive full credit. Your work should be mathematically correct and carefully and legibly written.

(6) A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.

(7) For problems 1–12, please write your choice of these multiple choice problems in the following table. This will make the work of the graders easier.
Problem 1. (15 points) Let \( z = x^2 - 3y^2 + 20 \), where \( x = 2 \cos t \) and \( y = 2 \sin t \). The value of \( \frac{dz}{dt} \) at \( t = \frac{\pi}{4} \) is ( ).

A. -4  
B. -8  
C. -16  
D. 2  
E. 6

Problem 2. (15 points) The directional derivative of \( f(x, y) = 3 - \frac{x^2}{10} + \frac{3y^2}{10} \) at the point \( (3, -1) \) in the direction of the vector \( (3, 4) \) is ( ).

A. \(-\frac{39}{10}\)  
B. \(-\frac{23}{10}\)  
C. \(-\frac{29}{30}\)  
D. \(-\frac{30}{40}\)  
E. \(-\frac{30}{50}\)
Problem 3. (15 points) Evaluate the following integral after reversing the order of integration.
\[ \int_{0}^{\sqrt{\pi}} \int_{\sqrt{\pi}}^{\pi} \sin(x^2) \, dx \, dy = (\quad). \]

A. 0
B. 1
C. \pi
D. 2
E. \pi^2

Problem 4. (15 points) Use a double integral to calculate the area of the region \( R \) bounded by
\( y = x^2 \) and \( x + y = 12 \). The area of \( R \) is (\quad).

A. 12
B. \frac{37}{4}
C. 25
D. \frac{45}{2}
E. \frac{343}{6}
Problem 5. (15 points) Use a double integral to calculate the volume of the region $E$ below the surface $z = xy + 10$ and above the region in the $xy$-plane bounded by $x^2 + y^2 = 4$ and by $x^2 + y^2 = 16$. The volume of $E$ is ( ).

A. $60\pi$
B. $120\pi$
C. $180\pi$
D. $240\pi$
E. $300\pi$

Problem 6. (15 points) Let $D$ be the region in the $xy$-plane that is bounded by $y = \sqrt{x}, y = 0$ and $x = 1$. Suppose that at each point $(x, y)$ in $D$ the density is given by $\rho(x, y) = x$. The center of mass of $D$ is then ( ).

A. ($\frac{1}{2}, \frac{1}{2}$)
B. ($\frac{3}{8}, \frac{3}{4}$)
C. ($\frac{1}{4}, \frac{5}{8}$)
D. ($\frac{3}{4}, \frac{7}{16}$)
E. ($\frac{5}{7}, \frac{5}{12}$)
Problem 7. (15 points) Let $D$ be the solid cone bounded by $z = 4 - \sqrt{x^2 + y^2}$ and $z = 0$. If the density of $D$ is the constant $K$, then the center of mass of $D$ is ( ).

A. $(0, 0, \frac{1}{2})$
B. $(0, 0, 1)$
C. $(0, 0, \frac{3}{2})$
D. $(0, 0, 2)$
E. $(0, 0, \frac{5}{2})$

Problem 8. (15 points) Let $D$ be the region between the spheres of radius 1 and 2 centered at the origin. Then

$$\iiint_D \frac{1}{(x^2 + y^2 + z^2)^{3/2}} \, dV = ( ).$$

A. 0
B. $2\pi$
C. $4\pi \ln 2$
D. 24
E. $\sqrt{3}$
Problem 9. (15 points) Let \( C \) be the curve given by \( \mathbf{r}(t) = (t, 3 \cos t, 3 \sin t) \), where \( 0 \leq t \leq \pi \). Then
\[
\int_{C} yz \cos x \, ds = ( ) .
\]
A. 4  
B. \( 4\sqrt{10} \)  
C. 6  
D. \( 6\sqrt{10} \)  
E. 8

Problem 10. (15 points) Let \( C \) be the line segment from \((1,0,-1)\) to \((3,4,2)\). Then
\[
\int_{C} xy \, dx + y^2 \, dy + yz \, dz = ( ) .
\]
A. 0  
B. 45  
C. \( \frac{110}{3} \)  
D. \( \frac{74}{5} \)  
E. \( \frac{99}{4} \)
Problem 11. (15 points) Let $C$ be the circle $x^2 + y^2 = 4$ with counterclockwise orientation. Use Green's Theorem to evaluate

$$\int_C x^2 y \, dx - xy^2 \, dy = ( \quad ).$$

A. 0  
B. $4\pi$  
C. $-4\pi$  
D. $8\pi$  
E. $-8\pi$

Problem 12. (15 points) Let $S$ be the part of the surface $z = 25 - x^2 - y^2$ that lies inside of the cylinder $x^2 + y^2 = 2$. The area of $S$ is $( \quad )$.

A. $\frac{17\sqrt{17} \pi}{8}$  
B. $\frac{9}{2} \pi$  
C. $\frac{14}{3} \pi$  
D. $\frac{3}{8} \pi$  
E. $\sqrt{2} \pi$
Problem 13. (25 points) Let

\[ f(x, y) = x^3 + y^3 - 9xy. \]

(a) (10 points) Find the critical points of \( f(x, y) \).

(b) (15 points) Use the Second Derivative Test to determine whether each critical point corresponds to a local maximum, local minimum, or saddle point.
Problem 14. (25 points) Let $S$ be the graph of the equation

$$xz^2 + z^3y^2 + y^2 = 46.$$ 

Find the equation of the tangent plane to $S$ at the point $(2, 1, 3)$. 
Problem 15. (30 points) Use the Fundamental Theorem for line integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle 4x^3y^2 - 2xy^3, 2x^4y - 3x^2y^2 + 4y^3 \rangle$, and the curve $C$ is defined by

$$r(t) = \langle t + \sin(\pi t), 2t + \cos(\pi t) \rangle, \quad 0 \leq t \leq 1.$$
Problem 16. (25 points) Find the mass of the sphere \( x^2 + y^2 + z^2 = 1 \) if its density at each point is given by \( \rho(x, y, z) = K y^2 \) where \( K \) is a constant.
Problem 17. (30 points) Let $S$ be the part of the graph of $z = x^2 + xy - y^2$ which lies inside of the cylinder $x^2 + y^2 = 1$. Find the surface integral $\iint_S \sqrt{1 + 5x^2 + 5y^2} \, dS$. 
Problem 18. (30 points) Let $S$ be the part of the paraboloid $z = x^2 + y^2$ below the plane $z = 1$ with upward orientation. Let $\mathbf{F}(x, y, z) = \langle x^2, xy, z \rangle$. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$. 
Problem 19. (30 points) Let $S$ be the part of the sphere $x^2 + y^2 + z^2 = 25$ which lies above the $xy$-plane. Let $F(x, y, z) = (y^2 + z^2, x + zy^3, xe^z + x^2)$. Use Stokes's Theorem to evaluate $\iint_S \text{curl} \ F \cdot d\mathbf{S}$. 
Problem 20. (25 points) Let $S$ be the unit sphere $x^2 + y^2 + z^2 = 1$. Let $\mathbf{F}(x, y, z) = \langle 3xy^2, 3yx^2, z^3 \rangle$. Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$. 