READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 13 pages, including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information and sign your name. Put your initials on the top of every page, in case the pages become separated. Books and notes are NOT ALLOWED. NO CALCULATORS ARE ALLOWED. Do your work in the blank spaces and back of pages of this booklet and say where your work is, if this is not clear. Show all your work.

There are 15 machine-graded problems worth 7 points each, for a total of 105 points. There are 7 hand-graded problems worth a total of 95 points. The total for the entire exam is 200 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-15):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle on the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong.

INSTRUCTIONS FOR THE HAND-GRADED PART. (Questions 16-22):
SHOW ALL WORK. Unsupported answers will receive little credit.

The last two pages of the exam are provided as scratch. Do not detach them.

Notice regarding the machine-graded sections of the exam: Either the student or the School of Mathematics may request a regrade of the machine-graded part. All the regrades will be based on responses in the test booklet, and not on the machine-graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. Therefore, work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of the booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing DOWN. Have your ID card in your hand when turning in your exam.

Machine-graded part ______  Hand-graded part ______

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<thead>
<tr>
<th>Problem</th>
<th></th>
</tr>
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<tbody>
<tr>
<td>16</td>
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<td>Subtotal</td>
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Total ______

Letter Grade ______
PART I: MACHINE-GRADED PROBLEMS

1. Let

\[ f(x) = \begin{cases} 
\frac{x^2 - 1}{x + 1} & \text{if } x \leq -7, \\
\frac{x^2 + 3x - 28}{x + 7} & \text{if } x > -7.
\end{cases} \]

Then \( \lim_{x \to (-7)^+} f(x) \) is equal to

(A) \(-7\)

(B) \(-8\)

(C) \(7\)

(D) \(-11\)

(E) does not exist

2. Find all of the vertical and horizontal asymptotes for \( f(x) = \frac{(x^2 + 1)(x + 3)}{(x - 3)^2(x + 1)} \).

(A) \(x = 3\) and \(x = -1\)

(B) \(x = 1, x = -3\) and \(y = 1\)

(C) \(x = 1, \text{and } x = -3\)

(D) \(x = 3, x = -1\) \(y = 1\) and \(y = 0\)

(E) \(x = 3, x = -1\) and \(y = 1\)

3. \( \frac{d}{dx} \left[ \left( \ln x \right)^{1/x} \right] \) is equal to

(A) \(1\)

(B) \(\frac{\ln x}{x^2}\)

(C) \(\frac{\ln x}{x^2} \left( \frac{1}{\ln x} - \ln(\ln x) \right)\)

(D) \(\frac{\ln x}{x^2} \left( \frac{1}{\ln x} + \ln(\ln x) \right)\)

(E) None of the above
4. \[
\frac{d}{dx} \left( \sin \left( \frac{3}{x^2 + 1} \right)^2 \right)
\] is equal to

(A) \(-\frac{6x}{(x^2+1)^2} \sin \left( \frac{3}{x^2+1} \right) \cos \left( \frac{3}{x^2+1} \right)\)

(B) \(\frac{6x}{(x^2+1)^2} \sin \left( \frac{3}{x^2+1} \right) \cos \left( \frac{3}{x^2+1} \right)\)

(C) \(\frac{12x}{(x^2+1)^2} \sin \left( \frac{3}{x^2+1} \right)\)

(D) \(-\frac{12x}{x^2+1} \sin \left( \frac{3}{x^2+1} \right) \cos \left( \frac{3}{x^2+1} \right)\)

(E) \(-\frac{12x}{(x^2+1)^2} \sin \left( \frac{3}{x^2+1} \right) \cos \left( \frac{3}{x^2+1} \right)\)

5. Let \(f(x) = \frac{1}{\sqrt{x^2 + 1}}.\) Then \(f'(2)\) is equal to

(A) \(-\frac{2}{9}\)

(B) \(\frac{2}{9}\)

(C) \(-\frac{2}{27}\)

(D) \(-\frac{4}{9}\)

(E) \(-\frac{1}{54}\)

6. Estimating \(\arctan(0.98)\) by the method of linear approximations (i.e., by differentials) we get

(A) \(\frac{\pi}{4} - \frac{1}{550}\)

(B) \(\frac{\pi}{4} - \frac{1}{100}\)

(C) \(\frac{\pi}{4} + \frac{1}{100}\)

(D) \(\frac{\pi}{4} + \frac{1}{50}\)

(E) \(\frac{\pi}{4(1+(1/50)^2)}\)
7. The radius of a right circular cylinder of height 20 centimeters increases at the rate of 2 centimeters per second. At what rate does the volume of the cylinder increase at the instant when the volume is \(80\pi\) cubic centimeters? Express your answer in cubic centimeters per second.

(A) \(80\pi\)

(B) \(160\sqrt{\pi}\)

(C) \(160\pi\)

(D) 160

(E) None of the above

8. 

\[ \lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^{-n/3} = \]

(A) \(e\)

(B) 1

(C) \(e^{1/3}\)

(D) \(e^{-1/3}\)

(E) 0

9. Let 

\[ f(x) = \int_0^{2^{1/3}} x \frac{dt}{\sqrt{t^3 + 13}} \]

Then 

\[ f'(1) = \]

(A) \(\frac{1}{\sqrt{14}}\)

(B) \(\frac{3^{1/3}}{4}\)

(C) \(\frac{1}{4}\)

(D) \(\frac{3^{1/3}}{\sqrt{14}}\)

(E) none of the above
10. Let \( f(x) = x^3 - 3x^2 - 9x \) with domain \(-\infty < x < \infty\). Then

(A) \( f(x) \) has an absolute maximum at \( x = -1 \), and an absolute minimum at \( x = 3 \);

(B) \( f(x) \) has a local, but not absolute, maximum at \( x = -1 \), and a local, but not absolute minimum at \( x = 3 \);

(C) \( f(x) \) has a local, but not absolute, maximum at \( x = -1 \), and an absolute minimum at \( x = 3 \);

(D) \( f(x) \) has a local, but not absolute, minimum at \( x = -1 \), and a local, but not absolute maximum at \( x = 3 \);

(E) \( f(x) \) has a local, but not absolute, maximum at \( x = -1 \), and local, but not absolute minima at \( x = 0 \) and at \( x = 3 \).

11. Let \( f(x) = 5x^4 - x^5 \) with domain \(-\infty < x < \infty\). Then

(A) \( f(x) \) is concave upward on \((-\infty, 0)\) and on \((0, 3)\), and concave downward on \((3, \infty)\);

(B) \( f(x) \) is concave upward on \((-\infty, 3)\) and concave downward on \((3, \infty)\);

(C) \( f(x) \) is concave upward on \((0, 3)\) and concave downward on \((-\infty, 0)\) and \((3, \infty)\);

(D) \( f(x) \) is concave upward on \((-\infty, 0)\) and on \((3, \infty)\), and concave downward on \((0, 3)\);

(E) none of the above

12.

\[
\int_1^2 x^{-1/2}(1 + x + x^{1/2}) \, dx =
\]

(A) \( \frac{10}{3} \sqrt{2} - \frac{5}{3} \)

(B) \( \frac{5}{3} \sqrt{2} - \ln 2 \)

(C) \( \frac{5}{3} - \frac{10}{3} \sqrt{2} \)

(D) \( 4\sqrt{2} - \frac{5}{3} \)

(E) \( \frac{7}{3} \sqrt{2} - \frac{5}{3} \)
13. \[ \int_0^1 \frac{x + 2}{x^2 + 1} \, dx = \]
(A) \(\arctan 1\)
(B) \(\frac{1}{2} \ln 2\)
(C) \(\frac{1}{2}(\ln 2 + \pi)\)
(D) \(\ln 2 + \arctan 1\)
(E) \(\frac{1}{2} \ln 2 + \frac{\pi}{4}\)

14. \[ \int_0^{\ln x} \sin(e^{2t})e^{2t} \, dt = \]
(A) \(\cos x^2 - \cos 1\)
(B) \(-\cos x^2 - \cos 1\)
(C) \(-\frac{1}{2}(\cos x^2 - 1)\)
(D) \(\frac{1}{2}(\cos x^2 - \cos 1)\)
(E) \(-\frac{1}{2}(\cos x^2 - \cos 1)\)

15. Let \(R\) be the region bounded by the curves \(y = x^3 + x, \ y = 18 - x^3\), and the \(y\)-axis. (The curves meet at \(x = 2\).) Then the volume of the solid obtained by revolving \(R\) about the \(x\)-axis is given by
(A) \(\pi \int_0^2 [(18 - x^3) - (x^3 + x)]^2 \, dx\)
(B) \(\pi \int_0^2 [(18 - x^3)^2 - (x^3 + x)^2] \, dx\)
(C) \(\pi \int_0^2 [(x^3 + x) - (18 - x^3)]^2 \, dx\)
(D) \(\pi \int_0^2 [(x^3 + x)^2 - (18 - x^3)^2] \, dx\)
(E) \(\pi \int_0^2 [(18 - x^3)^2 + (x^3 + x)^2] \, dx\)
PART II: HAND-GRADED PROBLEMS

Be sure you clearly indicate the steps you use to solve each problem.

16. (15 pts) Compute

$$\lim_{x \to 0} \frac{\ln(x + 1) - x}{x^2}.$$ 

Enter your final answer in the oval.

17. (10 pts) Let $x_1$ be an approximation of the root of $e^{-x} - x = 0$. Use Newton’s method to set up the expression for the next approximation $x_2$, in terms of $x_1$. 


18. (10 points) Let \( f(x) = x^5 + 12x - 6 \). Since \( f(0) < 0 \) and \( f(1) > 0 \), it follows that the function must have at least one real root \( r_1 \), i.e., \( f(r_1) = 0 \), for \( 0 < r_1 < 1 \). Show that \( f \) cannot have a second root anywhere on the real line. That is, show that there cannot be 2 distinct real numbers \( r_1, r_2 \) such that \( f(r_1) = f(r_2) = 0 \).

19. (10 pts) Compute

\[
\lim_{n \to \infty} \frac{\pi}{n} \sum_{i=1}^{n} \sin\left(\frac{\pi i}{n}\right).
\]
20. (15 pts) Find $\frac{dy}{dx}$ at the point $(x, y) = (\pi, \frac{\pi}{2})$ on the curve $\sin x + \cos y = \sin x \cos y$. 
21. (15 pts) Find the area enclosed by the curve \( y = |x^3 - x^2 - 2x| \) and the \( x \)-axis, between the lines \( x = -2 \) and \( x = 2 \).
22. (20 pts) A manufacturer of cylindrical cans (including top and bottom) receives a very large order for cans of volume \(2\pi \text{ ft}^3\). What radius and height will minimize the total surface area of such cans, and therefore the amount of metal needed to manufacture them? Recall that the lateral surface area of a cylinder of radius \(r\) and height \(h\) is \(2\pi rh\).