Last Name: ___________________________  First Name: ___________________________

Student ID#: _______________  Discussion Section: _______________  TA: _______________

I certify that the answers on this exam are my own, produced in accordance with all University and Institute of Technology policies on Scholastic Conduct.
Signature: ________________________________________________________________

Read and Follow These Instructions
The TAs should announce the number of pages on this exam. Check to see if any are missing. PRINT all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. You are not allowed to have any sort of books, notes, cell phones, calculators, computers, or any other electronic devices outside of your backpack in the exam room.

There are 15 machine-graded problems worth 10 points each and 6 hand-graded problems worth 25 points each for a total of 300 points. You have 3 hours to do the problems.

Instruction for Machine-graded Part (Questions 1-15):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. **Do not make any stray marks on the answer sheet.** When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. **Notice regarding the machine graded sections of this exam.** Either the student or the School of Mathematics may for any reason request a regrading of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. **Therefore, work and answers must be clearly shown on the test booklet**

Instructions for the hand-graded part (Questions 16-21):
You must show all steps in your solutions and make your reasoning clear with English sentences to earn credit. Simplification of answers not required. SHOW ALL WORK.

After you finish both parts of the exam: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing DOWN. Have your ID card in your hand when turning in your exam.
Total ______

Letter Grade ______

Multiple choice part ______ Hand-graded part ______
1. Find \( \frac{d}{dx}(x^2e^x - 3x^4 + 7)^{12} \).
   
   (A) \((2xe^x - 12x^3)^{12}\)
   (B) \((2xe^x + x^2e^x - 12x^3 + 7)^{12}\)
   (C) \((2xe^x + x^2e^x - 12x^3 + 7)^{11}\)
   (D) \(12(2xe^x + x^2e^x - 12x^3 + 7)^{11}\)
   (E) \(12(x^2e^x - 3x^4 + 7)^{11}(2xe^x + x^2e^x - 12x^3)\)

2. Compute \( \frac{d}{dx}\sqrt{\frac{2x + 4}{x - 1}} \)

   (A) \( \frac{1}{2} \left[ \frac{2x + 4}{x - 1} \right]^{-1/2} \left[ \frac{-6}{(x - 1)^2} \right] \)
   (B) \( \frac{1}{2} \left[ \frac{2x + 4}{x - 1} \right]^{-1/2} \left[ \frac{6}{(x - 1)^2} \right] \)
   (C) \( \frac{1}{2} \left[ \frac{2x + 4}{x - 1} \right]^{-1/2} \left[ \begin{array}{c} 2 \\ 1 \end{array} \right] \)
   (D) \( \sqrt{\frac{2}{1}} \)
   (E) none of the above
3. Compute \( \frac{d}{dx}[(\cos x)^{(\ln x)}] \)

(A) \((-\sin x)^{(1/x)}\)
(B) \([(\cos x)^{(\ln x)}][\frac{1}{x}(\ln(\cos x)) - (\ln x)(\tan x)]\)
(C) \([(\cos x)^{(\ln x)}][\frac{1}{x}(\ln x)(\cos x) - (\ln(\tan x))\]
(D) \([\ln x][(\cos x)^{(\ln x) - 1}][-\sin x]\)
(E) \([\ln x][(\cos x)^{(\ln x) - 1}\]

4. Let \( f(x) = x^3 - 6x^2 \). Then

(A) \( f(x) \) has local, but not absolute, minima at \( x = 0 \) and at \( x = 4 \);

(B) \( f(x) \) has local, but not absolute, maxima at \( x = 0 \) and at \( x = 4 \);

(C) \( f(x) \) has a local, but not absolute, maximum at \( x = 0 \), and an absolute minimum at \( x = 4 \);

(D) \( f(x) \) has a local, but not absolute, maximum at \( x = 0 \), and a local, but not absolute minimum at \( x = 4 \);

(E) \( f(x) \) has a local, but not absolute, maximum at \( x = 4 \), and a local, but not absolute, minimum at \( x = 0 \).
5. An isosceles right triangle is growing. ("Isosceles" means two of its sides have the same length; the hypotenuse is longer.) At the moment when its hypotenuse is $\sqrt{2}$ inches in length, its area is increasing at a rate of 2 square inches per second. At that moment, what is the rate of change in the length of one of its sides, in inches per second.

(A) 2
(B) $2\sqrt{3}$
(C) $3\sqrt{2}$
(D) $\sqrt{2}$
(E) $2\sqrt{2}$
6. The tangent line to the curve $x^5 + y^5 = 33$ at the point $(2, 1)$ has equation

(A) $y - 1 = (1/16)(x - 2)$
(B) $y - 1 = 16(x - 2)$
(C) $y - 1 = -16(x - 2)$
(D) $y - 1 = -(1/16)(x - 2)$
(E) $y - 1 = 32(x - 2)$
7. Let \( f(x) = x^3 - 6x^2 \). Then
(A) \( f(x) \) is concave downward on \((-\infty, 0)\) and concave upward on \((0, \infty)\);
(B) \( f(x) \) is concave downward on \((-\infty, 2)\) and concave upward on \((2, \infty)\);
(C) \( f(x) \) is concave upward on \((-\infty, 0)\) and concave downward on \((0, \infty)\);
(D) \( f(x) \) is concave upward on \((-\infty, 2)\) and concave downward on \((2, \infty)\);
(E) \( f(x) \) is concave upward everywhere.

8. The radius \( r \) of a sphere was measured and found to be 21 cm, with a possible error of at most 0.05 cm. Estimate the error in the area \( A = 4\pi r^2 \) of the sphere, using differentials.

(A) \( 4\pi(0.05)^2 \)
(B) \( 4\pi(21)^2 \)
(C) \( 8\pi(0.05) \)
(D) \( 8\pi(21)(0.05) \)
(E) \( \frac{4}{3}\pi(21)^3 \)
9. Let \( f(x) = x^3 - 7 \). If 2 is an initial guess of a root of \( f \), find the next guess via Newton's method.

(A) \( \frac{23}{12} \)

(B) \( \frac{22}{12} \)

(C) \( \frac{21}{12} \)

(D) \( \frac{20}{12} \)

(E) \( \frac{19}{12} \)
10. Compute \( \lim_{x \to 0} (1 + (\sin x))^{\cot x} \).

(A) \(1/e^2\)

(B) \(1/e\)

(C) 1

(D) \(e\)

(E) \(e^2\)
11. Find $\int_0^2 xe^{-x^2/2} \, dx$.

(A) 0
(B) $1 - (1/e^2)$
(C) $e - (1/e)$
(D) $e^2 - (1/e)$
(E) $-1 - e$

12. Find $\int_0^1 \frac{1}{\sqrt{1 - x^2}} \, dx$.

(A) 1
(B) $\pi/2$
(C) $\pi/3$
(D) $\pi/4$
(E) $\pi/6$
13. Let \( f \) be a one-to-one function, and assume that \( f(5) = 7 \) and that \( f'(5) = 2 \). Let \( g \) be the inverse of \( f \), so that \( g(f(x)) = x \). Find \( g(7) \) and \( g'(7) \). (Hint: Let \( y = f(x) \), so \( g(y) = x \). Evaluate \( g(y) = x \) at \( x = 5, y = 7 \). Then implicitly differentiate the equation \( g(y) = x \), and evaluate the resulting equation at \( x = 5, y = 7, y' = 2 \).)

(A) \( g(7) = 1/5 \) and \( g'(7) = 1/2 \)

(B) \( g(7) = 5 \) and \( g'(7) = 2 \)

(C) \( g(7) = 1/5 \) and \( g'(7) = 2 \)

(D) \( g(7) = 5 \) and \( g'(7) = 1/2 \)

(E) cannot be determined from the given information

14. The substitution \( x = u^2 \) turns \( \int_{3}^{7} \cot \sqrt{x} \, dx \) into

(A) \( \int_{9}^{49} \cot u \, du \)

(B) \( \int_{9}^{49} 2u \cot u \, du \)

(C) \( \int_{\sqrt{3}}^{\sqrt{7}} \cot u \, du \)

(D) \( \int_{\sqrt{3}}^{\sqrt{7}} \frac{u \cot u}{2} \, du \)

(E) \( \int_{\sqrt{3}}^{\sqrt{7}} 2u \cot u \, du \)
15. Let \( f(x) = \int_{1}^{4x^3} e^{-t^2} \, dt \). Then \( f'(1) \) is equal to

(A) \( e^{-4} \)

(B) \( e^{-12} \)

(C) \( 12e^{-16} \)

(D) \( -4e^{12} \)

(E) \( 4e^{-3} \)
16. Define $y$ implicitly by $(\sin x) + (\cos y) = 1$.
   
   A. Find $y'$ as a function of $x$ and $y$.
   
   B. Find $y''$ as a function of $x$ and $y$.
   
   (Take care that your final answer does not involve $y'$.)
17. Let $S$ be the solid obtained by rotating, about the $x$-axis, the region bounded by $y = 1 - x^2$ and $y = 0$.

(A) using the method of cylindrical shells, set up an integral (or sum of integrals) to express the volume of $S$.

(B) using the method of slices (or "disk method"), set up an integral (or sum of integrals) to express the volume of $S$.

(C) Choose (B) to evaluate the volume of $S$. 
18. Let \( f(x) = x^3 - 9x^2 + 24x + 7 \).
   
   On which intervals is \( f(x) \)

   a) increasing

   b) decreasing

   c) concave up

   d) concave down
19. Compute \( \lim_{x \to \infty} \sqrt{x^2 + 6x + 4} - x. \)
20. A large ship in the ocean is one mile directly north of a small boat. The boat is stationary and the ship is traveling along a straight line that will eventually carry it to a point five miles directly east of the boat. How close does the ship get to the boat?
21. A kite is connected by string to a stationary child who is letting out more string over time, hoping the kite will gain altitude. When 50 feet of string have been let out, the kite is at an altitude 30 feet above the altitude of the child’s hand, and it’s moving 2 feet per second straight upward. At what rate is the angle between the string and the horizontal increasing at that moment, in radians per second? (Assume that the string always follows a straight line to the kite.)