Math 1271
Spring, 2010
FINAL EXAM

T.A. Instructor______________________ Discussion Section_______ I.D.#_____

READ AND FOLLOW THESE INSTRUCTIONS
This booklet contains 19 pages, including this cover page and 2 blank pages at the end. Check to see if any are missing. PRINT at the top of this page all the requested information, and sign your name. Put your initials on the top of every page, in case the pages become separated. Books, notes and calculators are NOT PERMISSIBLE. Do your work in the blank spaces, back sides of pages and the blank pages at the end of this booklet. Show all your work.

There are 10 machine-graded problems worth 10 points each and 10 hand-graded problems worth 15 points each, together for a total of 250 points.

INSTRUCTIONS FOR MACHINE-GRADED PART (Problems 1-10):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong.

INSTRUCTIONS FOR THE HAND-GRADED PART (Problems 11-20):
SHOW ALL WORK. Unsupported answers will receive little credit.

Notice regarding the machine graded sections of this exam. Either the student or the School of Mathematics may for any reason request a regrading of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked "GENERAL PURPOSE ANSWER SHEET" facing DOWN. Have your ID card in your hand when turning in your exam.

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Subtotal  

1-10  

Total

Letter Grade _______
MULTIPLE CHOICE PORTION, Problems 1-10. There is only one correct answer for each problem.

**Problem 1.** Indicate all of the points on the real line \((-\infty, \infty)\) where the following function is discontinuous:

\[
    f(x) = \begin{cases} 
        e^x & \text{if } x \leq 0 \\
        \sin(x) & \text{if } 0 < x < \frac{\pi}{4} \\
        \cos(x) & \text{if } x \geq \frac{\pi}{4}
    \end{cases}
\]

(A) \(x = 0\)

(B) \(x = \pi/4\)

(C) \(x = 0, \ x = \pi/4\)

(D) \(x = \ln(\pi/4)\)

(E) No points of discontinuity.

**Problem 2.** Find the derivative of the function \(f(x) = (x+1)^4(x-1)^7\) at the point \(x = 0\).

(A) 1

(B) 2

(C) 3

(D) 4

(E) 5
Problem 3. Find the area of the region between the graph of the function \( f(x) = (1 - \sqrt{x})^2 \) and the x-axis from \( x = 0 \) to \( x = 1 \)

(A) \( \frac{1}{6} \)

(B) \( \frac{1}{3} \)

(C) \( \frac{1}{2} \)

(D) \( -\frac{1}{2} \)

(E) \( -100 \)

Problem 4. Find an absolute maximum value of the function \( f(x) = 2 \ln x + 3x - x^2 \) on the interval \((0, \infty)\).

(A) \( \ln 2 \)

(B) \( 2\ln 2 \)

(C) \( \ln 2 + 2 \)

(D) \( 2\ln 2 + 2 \)

(E) No such maximum value exists.
Problem 5. Compute
\[
\frac{d}{dx} \sqrt{\frac{2x + 4}{x - 1}}.
\]

(A) \[\frac{1}{2} \left( \frac{2x + 4}{x - 1} \right)^{-1/2} \left( \frac{6}{(x - 1)^2} \right)\]

(B) \[\frac{1}{2} \left( \frac{2x + 4}{x - 1} \right)^{-1/2} \left( \frac{-6}{(x - 1)^2} \right)\]

(C) \[\frac{1}{2} \left( \frac{2x + 4}{x - 1} \right)^{-1/2}\]

(D) \[\sqrt{2}\]

(E) None of the above.
Problem 6. Find
\[ \int_0^1 \frac{2}{1 + x^2} dx. \]

(A) 1

(B) \( \pi/4 \)

(C) \( 2\pi/3 \)

(D) \( \pi/2 \)

(E) \( \pi/3 \)

Problem 7. The graph of the function
\[ f(x) = \frac{1 + 4x^2}{1 - 4x^2} \]
has the following vertical and horizontal asymptotes (list all of them):

(A) \( x = 0, y = 1 \)

(B) \( x = 0, y = -1 \)

(C) \( x = 1/2, x = -1/2, y = 1 \)

(D) \( x = 1/2, x = -1/2, y = -1 \)

(E) No asymptotes
Problem 8. Find the point on the graph of the function \( f(x) = \sqrt{x}, \ x \geq 0 \), that is closest to the point \((1, 0)\).

(A) \(\frac{1}{4}, \frac{1}{2}\)

(B) \(\frac{1}{3}, \frac{1}{\sqrt{3}}\)

(C) \(\frac{1}{2}, \frac{1}{\sqrt{2}}\)

(D) \(\frac{3}{4}, \frac{\sqrt{3}}{2}\)

(E) None of the above.

Problem 9. Calculate the following limit.

\[
\lim_{n \to \infty} \left[ 1 + \sin \left( \frac{1}{n} \right) \right]^{3n}
\]

(A) 1

(B) \(\ln 2\)

(C) \(e^3\)

(D) \(\infty\)

(E) None of the above.
Problem 10. Let

\[ f(x) = x^{x^2+2}, \; x > 0. \]

Find \( f'(x) \).

(A) \( (x^2 + 2)x^{x^2+1} \)

(B) \( (2x)(x^2 + 2)x^{x^2+1} \)

(C) \( (\ln 2x)x^{x^2+2} \)

(D) \( \left( 2x \ln x + \frac{x^2 + 2}{x} \right) x^{x^2+2} \)

(E) \( (x^2 + 2)(\ln x)x^{x^2+2} \)
HAND-GRADED PORTION, Problems 11-20. Please show your work in order to get full credit.

Problem 11. Let
\[ f(x) = \sqrt{\frac{e^x + 1}{2}}. \]

(a) Linearize the function \( f \) at \( x = 0 \).

(b) Use your linearization to estimate \( f(0.1) \).
Problem 12. Let \( f(x) = -x^3 + x^2 + x + 3. \)

(a) Determine the intervals on which the graph of \( f(x) \) is increasing and decreasing.

(b) Determine the intervals on which the graph of \( f(x) \) is concave up and concave down.

(c) Determine the points on the graph where \( f(x) \) achieves local maximum and minimum values.

(d) Sketch the graph of \( f(x) \) on the interval \([-1, 2]\), accurately using the information that you have already determined.
Problem 13. Calculate
\[
\int_0^{\pi/2} (\cos^2 x - 2 \cos x) \sin x \, dx.
\]
Problem 14. Find the indefinite integral
\[ \int \left( \frac{2x + e^{-\sqrt{x+1}}}{\sqrt{x+1}} \right) \, dx. \]

Hint: Use the substitution \( u = \sqrt{x + 1} \).
Problem 15. Let
\[ g(x) = \int_1^x \ln(t^2 - 1) \, dt, \quad x \geq 1. \]

(a) Find the first and second derivatives of \( g \).

(b) Find the absolute maximum and minimum values of \( g \) on the interval \([1, 10]\).
Problem 16. Let $f(x) = x^3 + x + 3$.

(a) Show that the graph of $f(x)$ crosses the $x$-axis at exactly one point $(p, 0)$.

(b) Use $-1$ as an initial guess for the value of $p$, and then carry out one step of Newton’s method to find a new approximation for the value of $p$. 
Problem 17. The graph of the equation \( x = y^2 - 2y - 3 \) is a parabola that opens to the right. Draw a sketch of this parabola, and find the area of the region that is enclosed by the parabola and the two coordinate axes.
Problem 18. Calculate the following limit, if it exists. Otherwise, explain why it does not exist.

\[ \lim_{x \to 1} \frac{\ln(x)}{\cos(\frac{\pi}{2}x)} \]
Problem 19. The graph of the equation $x^2 - 2xy - y^2 - x = 2$ is a hyperbola. Verify that the point $(-1, 2)$ is on this hyperbola, and find the equation of the tangent line to the graph at that point.
Problem 20. The width of a rectangle is growing at the rate of 2 inches per minute, and its height is getting smaller at the rate of 3 inches per minute. Find the rate of change of the length of diagonal of the rectangle when the width is 20 inches and the height is 15 inches. (Note: when the width is 20 and the height is 15, the diagonal is 25 inches.)
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Name:__________________________________________

Scratch #1