This booklet contains 25 pages, including this cover page. Please print on the top all the requested information, and sign your name. Put your name on the top of every page, in case the pages become separated. There are 12 machine-graded problems and 8 hand-graded problems. Do not use calculators. No books or notes are permitted. No electronic devices are allowed.

For the machine-graded problems (1-12), please use a soft pencil (No. 1 or No 2) to answer them in the answer sheet, on which you must also carefully enter all the requested information according to the instructions. For the hand-graded problems (13-20), show all your work.

*Notice regarding the machine graded sections of this exam:* Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. Therefore work and answers must be clearly shown on the test booklet.

Multiple choice part (96 pst.) Hand-graded part (104 pst.)

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Total
PART I: Machine-Graded Problems. Detailed justification is not required.

(1) (8 pts.)
\[ \int \frac{dx}{2\sqrt{x}(x + 5\sqrt{x} + 6)} = \]

(a) \( \ln |\sqrt{x} - 2| - \ln |\sqrt{x} - 3| + C \)
(b) \( \ln |\sqrt{x} + 2| - \ln |\sqrt{x} + 3| + C \)
(c) \( \ln |\sqrt{x} + 3| + \ln |\sqrt{x} + 2| + C \)
(d) \( \arctan (\sqrt{x} + 6) + C \)
(e) \( \ln |\sqrt{x} + 3| - \ln |\sqrt{x} + 2| + C \)
(2) (8 pts.)
\[ \int_{0}^{\frac{\pi}{2}} \sin^4 x \cos^3 x \, dx = \]

(a) \( \frac{2}{35} \)
(b) \( \frac{1}{35} \)
(c) \( \frac{2}{35} \)
(d) \( \frac{2}{15} \)
(e) \( \frac{1}{15} \)
(3) (8 pts.)

\[ \int x \sin^2 x \, dx = \]

(a) \[ \frac{x^2}{4} + \frac{1}{4} x \sin (2x) - \frac{1}{8} \cos (2x) + C \]
(b) \[ \frac{x^2}{4} - \frac{1}{4} x \sin (2x) - \frac{1}{8} \cos (2x) + C \]
(c) \[ \frac{x^2}{4} - \frac{1}{4} x \sin (2x) + \frac{1}{8} \cos (2x) + C \]
(d) \[ \frac{x^2}{4} - \frac{1}{8} x \sin (2x) + \frac{1}{16} \cos (2x) + C \]
(e) \[ \frac{x^2}{4} + \frac{1}{16} x \sin (2x) + \frac{1}{8} \cos (2x) + C \]
(4) (8 pts.)
What is the area between the curves $R = \theta \cos \theta$ and $R = 3$ from $\theta = 0$ to $\theta = \frac{\pi}{6}$?

(a) $\int_{0}^{\frac{\pi}{6}} (3 - \theta \cos \theta)^2 d\theta$

(b) $\int_{0}^{\frac{\pi}{6}} (9 - \theta^2 \cos^2 \theta) d\theta$

(c) $\frac{1}{2} \int_{0}^{\frac{\pi}{6}} (3 - \theta \cos \theta)^2 d\theta$

(d) $\frac{1}{2} \int_{0}^{\frac{\pi}{6}} (9 - \theta^2 \cos^2 \theta) d\theta$

(e) None of the above.
(5) (8 pts.)

The improper integral \[ \int_{1}^{\infty} \frac{x \, dx}{(x^2 + 5)^3} \]

(a) has value 0
(b) has value \( \frac{\pi}{12} \)
(c) has value \( \frac{1}{144} \)
(d) is divergent
(e) has value \( \frac{1}{12} \).
(6) (8 pts.)

The constant solutions $y$ of the differential equation

$$t^2e^{2t}y''(t) - \sin (\ln t) \ y'(t)y(t) + e^{\arctan t}(y(t))^4 - 4e^{\arctan t} = 0$$

are

(a) $\sqrt{4}$
(b) $-\sqrt{2}$
(c) $-2\sqrt{2}$ and $2\sqrt{2}$
(d) $t + 1$ and $t + \sqrt{2}$
(e) $-\sqrt{2}$ and $\sqrt{2}$. 
(7) (8 pts.)
If \( x = t^2 + t \) and \( y = t^3 + 3t + 5 \), then \( \frac{d^2y}{dx^2} = \)
(a) \( \frac{6(t^2+t-1)}{(2t+1)^3} \)
(b) \( \frac{6t}{(2t+1)^3} \)
(c) \( \frac{6(t^2+t-1)}{(2t+1)^2} \)
(d) \( \frac{6t}{(2t+1)} \)
(e) \( 3t \)
(8) (8 pts.)
The parametric curve defined by \( x = t^3 - 3t \) and \( y = e^t - t \) has
(a) a vertical tangent at \((-2, e - 1)\) and \((2, \frac{e^2 - 1}{e})\)
(b) a vertical tangent at \((2, \frac{e^2 - 1}{e})\)
(c) a horizontal tangent at \((0, 1)\)
(d) 3 distinct \(y\)-intercepts
(e) All of the above statements are true.
(9) (8 pts.)
The exact value of
\[ \lim_{n \to \infty} \frac{\sqrt[3]{8n^8} + 5n^2 - 10n + 5}{n + \cos(9^n)} \]
is

(a) \infty
(b) \sqrt{5}
(c) 0
(d) 2
(e) The limit does not exist.
(10) (8 pts.)
Find the surface area obtained by rotating about the x-axis the portion of the curve $y = 4 - x^2$ in the first quadrant ($x \geq 0, y \geq 0$).

(a) $\pi \int_{0}^{2} (4 - x^2) \sqrt{1 + 4x^2} \, dx$

(b) $\pi \int_{0}^{4} x \sqrt{1 + 4x^2} \, dx$

(c) $2\pi \int_{0}^{2} (4 - x^2) \sqrt{1 + 4x^2} \, dx$

(d) $2\pi \int_{0}^{4} x \sqrt{1 + 4x^2} \, dx$

(e) None of the above.
(11) (8 pts.)
The Taylor polynomial of degree 2, \( T_2(x) \), centered at 0, of the function
\( f(x) = \frac{1}{\sqrt{1+x^2}} \) is
(a) \( 1 - \frac{1}{2}x^2 \)
(b) \( 1 - x^2 \)
(c) \( 1 + \frac{1}{2}x^2 \)
(d) \( \frac{1}{2}x^2 \)
(e) \( 1 + \frac{3}{2}x^2 \)
(12) (8 pts.)
The equation $2x^2 + 2y^2 + 2z^2 = 4x - 8y + 6$ represents a sphere with
(a) center $(-1, 2, 0)$ and radius $\sqrt{6}$
(b) center $(1, -2, 0)$ and radius $2\sqrt{2}$
(c) center $(-1, 2, 0)$ and radius $2\sqrt{2}$
(d) center $(1, -2, 0)$ and radius $\sqrt{6}$
(e) center $(-2, 1, 0)$ and radius $2\sqrt{2}$
PART II: Hand-Graded Problems. A result alone does \textbf{not} count. Clearly \textbf{mark} your final results by underlining, etc.

(13) (15 pts.)

Compute $\int \arctan (2\sqrt{x}) \, dx$. 
(14) (10 pts.) Apply the Trapezoidal Rule to approximate \( \int_{0}^{\frac{\pi}{2}} \sin(x^3) \, dx \) using \( n = 4 \). Write your answer as a sum of sine functions. Write down any general formula you are using.
(15) (15 pts.) Suppose that $y' = \frac{\cos{x}}{y^2}$ and $y(0) = 1$. What is $y$? Write as a function of $x$. 
(16) (20 pts.) Do the following series converge absolutely, conditionally, or do they diverge? Explain your reasoning, including what tests you are using.

(a) \[ \sum_{n=1}^{\infty} \frac{3^n a^2 (a + 1)^2 \ldots (a + n)^2}{(2n)!} \]

(b) \[ \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n} \]
(17) (10 pts.) Find the convergence radius of the power series
\[ \sum \arctan(n) 5^n (3x - 7)^n. \]
(18) (14 pts.) Use a power series expansion to compute $\int_0^1 xe^{x^3}dx$. 
(19) (19 pts.) Find the angle between the planes

\[ \alpha : 2x + y - 3z + 6 = 0 \]
\[ \beta : 4x + 2y + z + 5 = 0 \]

and the parametric equations of the line of intersection.
(20) (10 pts.) Find the plane that passes through \((5, 0, 7)\) and that is perpendicular to the line defined by the equation \(\frac{x+2}{2} = \frac{y-3}{3} = \frac{1-z}{6}\). Show your work.
SCRATCH I
SCRATCH II
SCRATCH III
Some Formulas

\[ \sin^2 x + \cos^2 x = 1 \]
\[ \tan^2 x + 1 = \sec^2 x \]
\[ \cot^2 x + 1 = \csc^2 x \]

\[ \sin (2x) = 2 \sin x \cos x \]
\[ \cos (2x) = \cos^2 x - \sin^2 x \]
\[ \sin^2 x = \frac{1}{2} [1 - \cos (2x)] \]
\[ \cos^2 x = \frac{1}{2} [1 + \cos (2x)] \]

\[ \sin A \cos B = \frac{1}{2} [\sin (A - B) + \sin (A + B)] \]
\[ \sin A \sin B = \frac{1}{2} [\cos (A - B) - \cos (A + B)] \]
\[ \cos A \cos B = \frac{1}{2} [\cos (A - B) + \cos (A + B)] \]

1) \[ \int \frac{dx}{x + a} = \ln |x + a| + C \]
2) \[ \int \frac{dx}{x^2 + a^2} = \frac{1}{a} \arctan \frac{x}{a} + C \]
3) \[ \int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left| \frac{x - a}{x + a} \right| + C \]
4) \[ \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a} + C \]
5) \[ \int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln \left| x + \sqrt{x^2 \pm a^2} \right| + C \]