READ AND FOLLOW THESE INSTRUCTIONS:
This booklet contains 17 pages, including this cover page. Check to see if any are missing. PRINT on the upper right-hand corner all the requested information, and sign your name. Put your name on the top of every page, in case the pages become separated. Do your work in the space provided in this booklet. There are three pages at the end of the exam that may be used as scratch, and a page with some trigonometric formulas. Show all your work. TEXTBOOKS, NOTES, AND CALCULATORS ARE NOT PERMISSIBLE.

INSTRUCTIONS FOR MACHINE-GRADED PART (Questions 1-15):
You MUST use a soft pencil (No. 1 or No. 2) to answer this part. Do not fold or tear the answer sheet, and carefully enter all the requested information according to the instructions you receive. DO NOT MAKE ANY STRAY MARKS ON THE ANSWER SHEET. When you have decided on a correct answer to a given question, circle the answer in this booklet and blacken completely the corresponding circle in the answer sheet. If you erase something, do so completely. Each question has a correct answer. If you give two different answers, the question will be marked wrong. There is no penalty for guessing, but if you don’t answer a question, skip the corresponding line in the answer sheet. Go on to the next question.

INSTRUCTIONS FOR THE HAND-GRADED PART (Questions 16-19):
Write your answers in the boxes which are provided. Show all work in the space provided below each problem. If you need extra space, state where the work is being done. Unsupported answers may receive little credit.

Notice regarding the machine graded sections of this exam: Either the student or the School of Mathematics may for any reason request a regrade of the machine graded part. All regrades will be based on responses in the test booklet, and not on the machine graded response sheet. Any problem for which the answer is not indicated in the test booklet, or which has no relevant accompanying calculations will be marked wrong on the regrade. Therefore, work and answers must be clearly shown on the test booklet.

AFTER YOU FINISH BOTH PARTS OF THE EXAM: Place the answer sheet between two pages of this booklet (make a sandwich), with the side marked “GENERAL PURPOSE ANSWER SHEET” facing DOWN. Have your ID card in your hand when turning in your exam.

Each multiple choice question counts 12 points, 180 points total. The total for the hand-graded questions of varying credit is 120 points. The total for the entire exam is 300 points.

Multiple choice part (points) _______ Hand-graded part (points) _______

Total (points)_______
1. $\int_{1}^{2} \frac{dx}{x(x+1)} =$

(A) $\frac{1}{3}$

(B) $\frac{2}{3}$

(C) $\ln{2} - \ln{3}$

(D) $\frac{4}{3}$

(E) $\ln{4} - \ln{3}$

2. $\int_{0}^{\pi/2} \cos^2{x} \sin^3{x} \, dx =$

(A) 1

(B) $\frac{1}{4}$

(C) $\frac{1}{5}$

(D) $\frac{2}{15}$

(E) none of the above.
3. Consider the improper integral

\[ \int_1^\infty \frac{1}{(3t + 1)^2} \, dt \]

(A) This improper integral converges and has value \( \frac{1}{4} \)

(B) This improper integral converges and has value \( \frac{1}{12} \)

(C) This improper integral converges and has value \( \frac{1}{16} \)

(D) This improper integral diverges and can be assigned no value.

(E) This improper integral diverges and has value \( \infty \).

4. Partial fraction decomposition of

\[ \frac{2x^3 + 2x^2 + 3x + 3}{x^2(x^2 + 3)} \]

should be looked for in the form

(A) \( \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^2 + 3} \)

(B) \( \frac{B}{x^2} + \frac{C}{x^2 + 3} \)

(C) \( \frac{B}{x^2} + \frac{Cx + D}{x^2 + 3} \)

(D) \( \frac{A}{x} + \frac{B}{x^2} + \frac{Cx + D}{x^2 + 3} \)

(E) none of the above.
5. A culture of bacteria grows according to the exponential law

\[ \frac{dP}{dt} = 0.05P \]

and satisfies the initial condition \( P(0) = 20 \). \((t \text{ is in minutes})\). Then \( P(40) \) equals (approximately)

(A) \( 800e \)
(B) \( 20e^{0.1} \)
(C) \( 20e^2 \)
(D) \( 0.05e^{20} \)
(E) 800

6. The area of the surface obtained by rotating the curve \( y = \frac{1}{3}x^3 \), \( 0 \leq x \leq 2 \) about the \( x\)-axis is given by

(A) \( \frac{2}{3} \pi \int_0^2 x^3 \sqrt{1 + x^4} \, dx \)
(B) \( 2\pi \int_0^2 x \sqrt{1 + x^4} \, dx \)
(C) \( \frac{2}{3} \pi \int_0^2 x^3 \sqrt{1 + x^2} \, dx \)
(D) \( \frac{2\pi}{3} \int_0^2 x^3 \, dx \)
(E) \( 2\pi \int_0^2 \sqrt{1 + x^2} \, dx \)
7. An equation of the tangent line to the curve \( x = 1 + 3t^2, \ y = 2t - \frac{1}{t} \) at the point corresponding to \( t = 1 \) is

(A) \( y = 4x \)
(B) \( y = 2t - 1 \)
(C) \( y = \frac{1}{2}x - 1 \)
(D) \( y = 2x - 7 \)
(E) none of the above.

8. The length of the curve \( x = \frac{1}{3}(2t + 1)^{3/2}, \ y = \frac{1}{2}t^2, \ 0 \leq t \leq 1 \) is given by

(A) \( \int_{0}^{1} \frac{1}{6}t^2(2t + 1)^{3/2} \, dt \)
(B) \( \int_{0}^{1} \left[ \frac{1}{9}(2t + 1)^3 + \frac{1}{4}t^4 \right] \, dt \)
(C) \( \int_{0}^{1} (t^2 + 2t + 1) \, dt \)
(D) \( \int_{0}^{1} (t + 1) \, dt \)
(E) none of the above.
9. The point \((2, 7\pi/6)\) in polar coordinates \((r = 2, \theta = 7\pi/6)\) is expressed in Cartesian coordinates by

(A) \((-1, -\sqrt{3})\)

(B) \((-\sqrt{3}, -1)\)

(C) \((1, -\sqrt{3})\)

(D) \((\sqrt{3}, 1)\)

(E) \((1, \sqrt{3})\)

10. The area between the polar curves \(r = \sin \theta\) and \(r = \cos \theta\) in the sector \(0 \leq \theta \leq \frac{\pi}{4}\) equals:

(A) \(\frac{1}{4}\)

(B) \(\frac{1}{8}\)

(C) \(\frac{\pi}{4}\)

(D) \(\frac{1}{2}\)

(E) 1
11. \( \lim_{n \to \infty} (n \cos(\frac{1}{n}) - n) = \)

(A) 1
(B) –1
(C) \( \frac{1}{2} \)
(D) \(-\frac{1}{2}\)
(E) 0

12. The radius of convergence and the interval of convergence of the power series

\[
\sum_{n=1}^{\infty} \frac{3^n x^n}{(n^2 + n)2^n}
\]

are:

(A) \( \frac{3}{2}, \left[-\frac{3}{2}, \frac{3}{2}\right] \)
(B) \( \frac{3}{2}, \left(-\frac{3}{2}, \frac{3}{2}\right) \)
(C) 1, \([-1, 1]\)
(D) \( \frac{2}{3}, \left[-\frac{2}{3}, \frac{2}{3}\right] \)
(E) \( \frac{2}{3}, \left(-\frac{2}{3}, \frac{2}{3}\right) \)
13. The volume of the parallelepiped determined by the vectors \( <1, 4, 6>, <2, 1, 0> \) and \( <7, -1, 4> \) is

(A) 96

(B) 82

(C) 68

(D) 42

(E) 0

14. The plane perpendicular to the segment with end points \( A(3, 1, -2) \) and \( B(7, 3, 4) \), and containing \( A \), has equation

(A) \( 2x + y + 3z = 29 \)

(B) \( 7(x - 3) + 3(y - 1) + 4(z + 2) = 0 \)

(C) \( 3(x - 7) + (y - 3) - 2(z - 4) = 0 \)

(D) \( 2x + y + 3z = 0 \)

(E) \( 2x + y + 3z = 1 \)
15. A particular helix (an ascending spiral) is traced by the vector function \( \mathbf{r}(t) = < 2 \sin 2t, 3t, -2 \cos 2t > \). What is the unit tangent vector \( \mathbf{T}(t) \) at the point \((0, 3\pi, 2)\)? (Recall that \( \mathbf{T}(t) = \frac{\mathbf{r}'(t)}{||\mathbf{r}'(t)||} \).)

(A) \( \frac{1}{5} < 4, 3, 0 > \)

(B) \( \frac{1}{5} < 0, 3, 4 > \)

(C) \( \frac{1}{5} < 0, 3\pi, 2 > \)

(D) \( \frac{1}{25} < 4, 3, 0 > \)

(E) \( \frac{1}{5} < -4, 3, 0 > \)
16. (38 pts) Find the solution of the differential equation

\[ y' = \frac{2x(1 + y^2)}{(x^2 + 1)} \]

satisfying the initial condition \( y(0) = 1 \).
17. (37 pts) Find the 2nd degree Taylor polynomial $T_2(x)$ around 0 of the function $f(x) = \ln(1 + e^x)$. 
18. (38 pts) For each of the two series

(a) \( \sum_{n=1}^{\infty} \frac{e^n}{n^2} \),  
(b) \( \sum_{n=1}^{\infty} \frac{n^2 + 2}{n^3} \)

decide whether the series converges or diverges.
19. (37 pts) Evaluate the integral

\[ \int_0^{\pi/2} x^2 \cos x \, dx \]
Some Formulas

\[ \cos 2x = \cos^2 x - \sin^2 x \]
\[ \cos^2 x + \sin^2 x = 1 \]
\[ \sin^2 x = \frac{1}{2} (1 - \cos 2x) \]
\[ \cos^2 x = \frac{1}{2} (1 + \cos 2x) \]
\[ \tan^2 x + 1 = \sec^2 x \]
Scratch Paper 1
Scratch Paper 2